

Atom-atom correlations in dissociation of a spatially inhomogeneous molecular condensate

Magnus Ögren

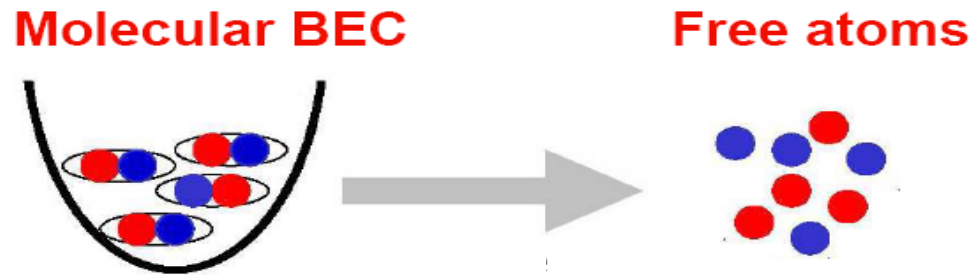
LTH, Lund

Stockholm, 1.st of April 2008

Thanks to:

**•Karen Kheruntsyan
at ACQAO/UQ in Brisbane**

Motivation to study dissociation into fermions:



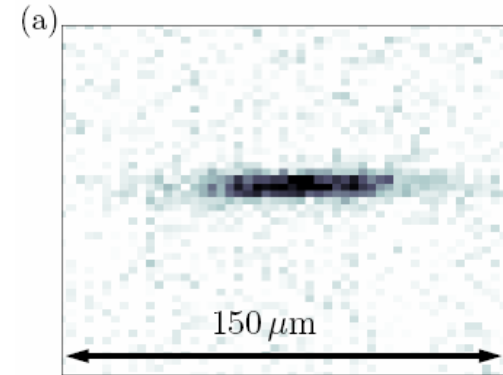
i) Conceptual:

Molecular dissociation as a fermionic analog of optical parametric down-conversion, a good candidate for developing the paradigm of *fermionic quantum atom optics*.

ii) Pragmatic:

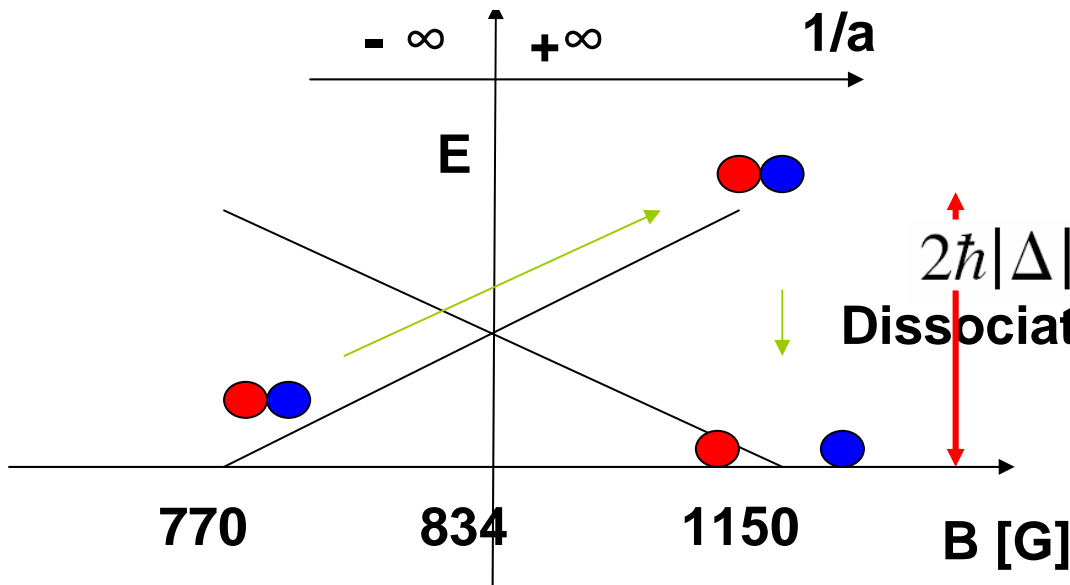
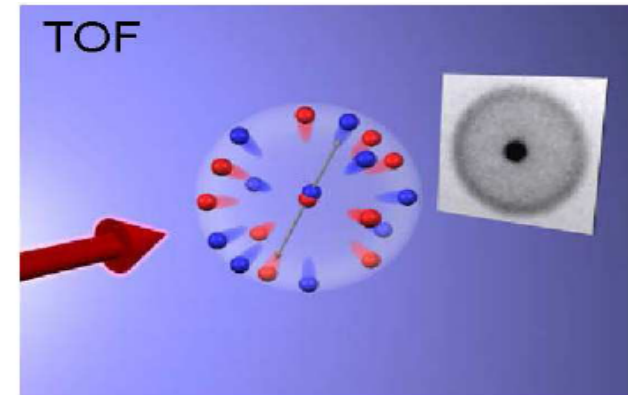
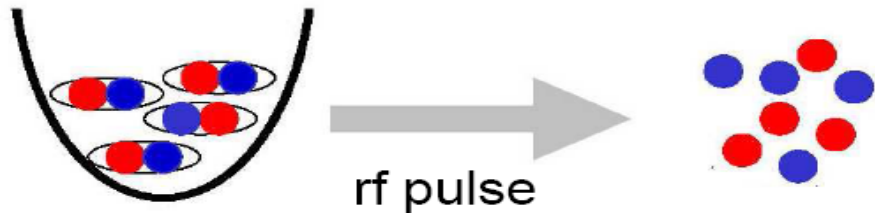
Can we explain the experimentally observed atom-atom correlations.

A few experiments world wide with condensed dimer molecules, e.g. at Swinburne Univ. of Tech. J. Fuchs *et al.*, arXiv:0709.2212v1



Molecular BEC

Free atoms

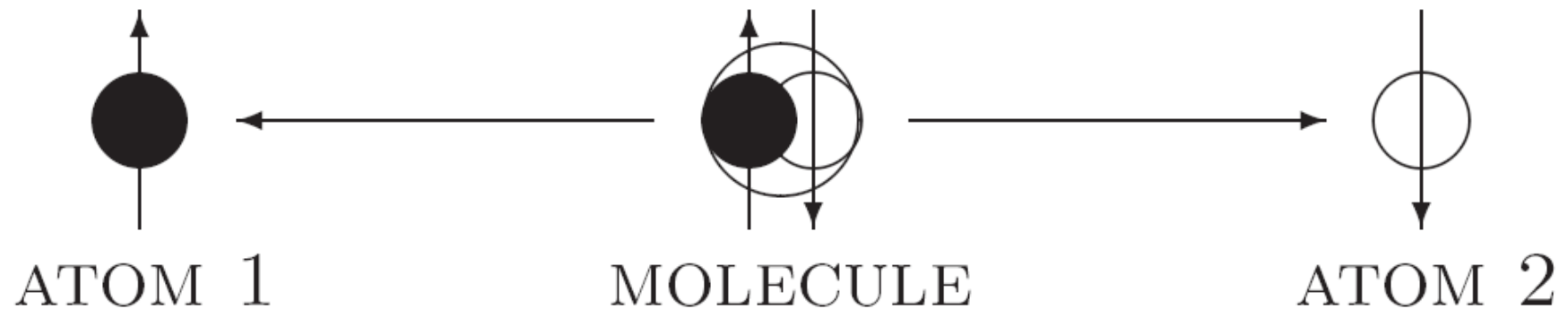


${}^6\text{Li}_2$: bosonic molecules

$$2\hbar|\Delta| \rightarrow \hbar^2(|\mathbf{k}_\uparrow|^2 + |\mathbf{k}_\downarrow|^2)/(2m)$$

${}^6\text{Li}$: fermionic atoms

Momentum conservation, source of correlation:



$$\mathbf{k}_\uparrow = -\mathbf{k}_\downarrow$$

Strongly correlated fermi gas

We quantify those correlations in a many-body system

Hamiltonian and approximations

$$\hat{H} = \hat{H}_0 - i\hbar\chi \int d\mathbf{x} \left(\hat{\Psi}_0^\dagger \hat{\Psi}_2 \hat{\Psi}_1 - \hat{\Psi}_1^\dagger \hat{\Psi}_2^\dagger \hat{\Psi}_0 \right)$$

Treat the molecules in a mean-field approximation:

$$g(\mathbf{x}) = \chi \left\langle \hat{\Psi}_0(\mathbf{x}, t = 0) \right\rangle$$

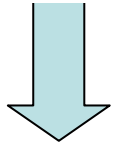
Results in a Hamiltonian quadratic in operators:

$$\hat{H} = \hat{H}_0 - i\hbar \int d\mathbf{x} g(\mathbf{x}) \left(\hat{\Psi}_2 \hat{\Psi}_1 - \hat{\Psi}_1^\dagger \hat{\Psi}_2^\dagger \right)$$

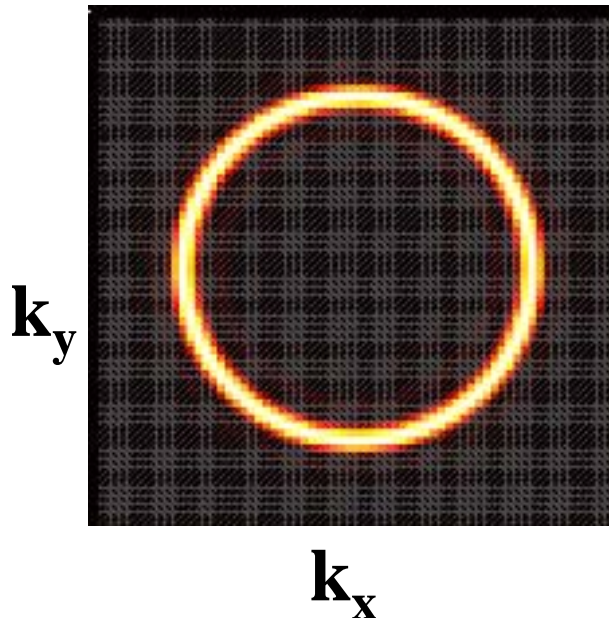
The corresponding Heisenberg equations of motion are then linear in plain wave base operators and can be solved with standard linear operator algebra methods.

Atomic densities:

$$\mathbf{k}_\uparrow = -\mathbf{k}_\downarrow$$



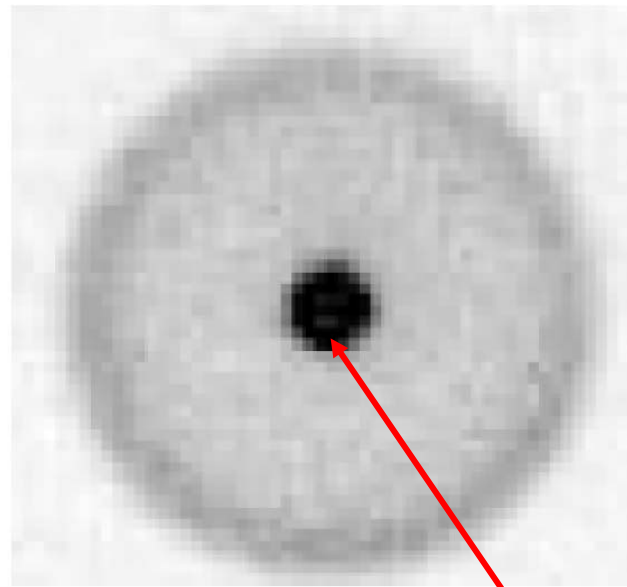
Best illustrated
with a 2D slice:



M. Greiner *et al.*, Phys. Rev. Lett. **94**, 110401 (2005) (JILA)

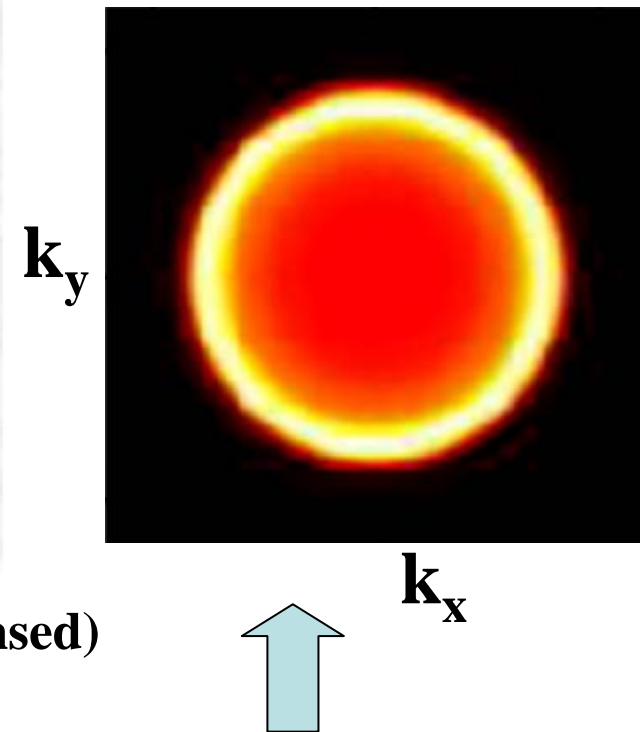


What experiment-
alists see (TOF):



(Atoms that never condensed)

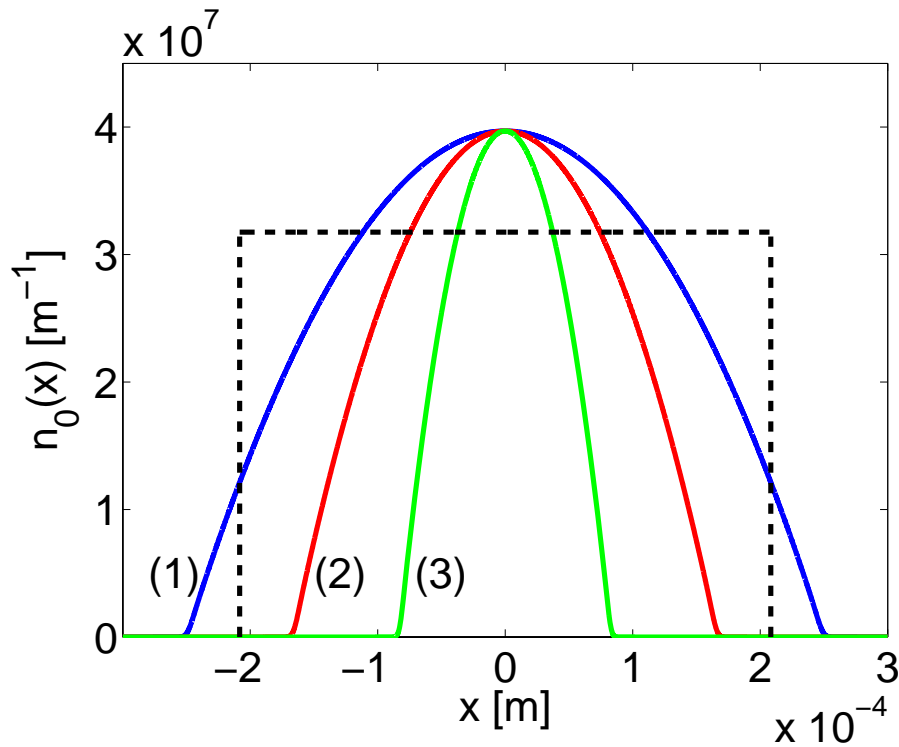
Theory: 'z-average'



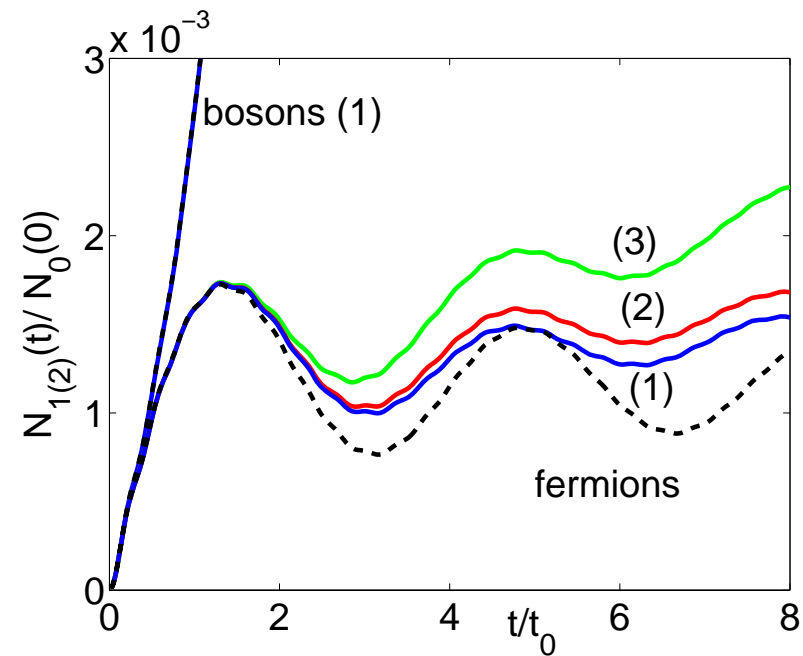
Uniform case can be solved analytically: Kheruntsyan, PRL **96**, 110401 (2006)

What is the effect of a non-uniform condensate?

Non-uniform case need to be solved numerically for each profile (1), (2), (3):



Not large effect on the number of atoms:

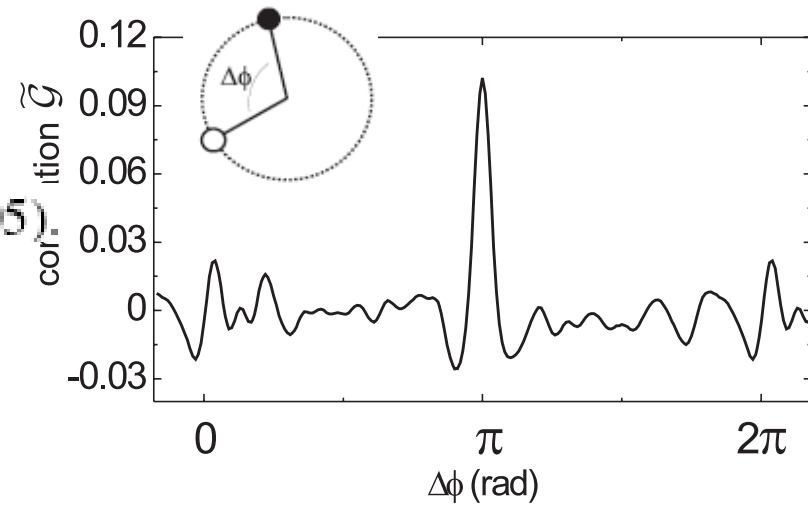


Where are the major effects ? Next slides...

Atom-atom pair correlations

Correlation measurement (TOF):

M. Greiner *et al.*, Phys. Rev. Lett. **94**, 110401 (2005)

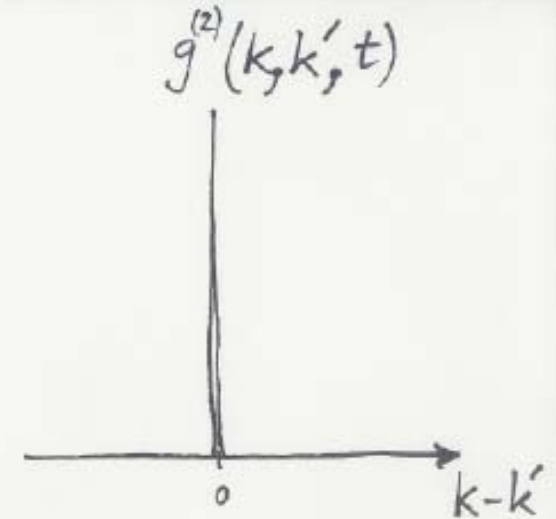
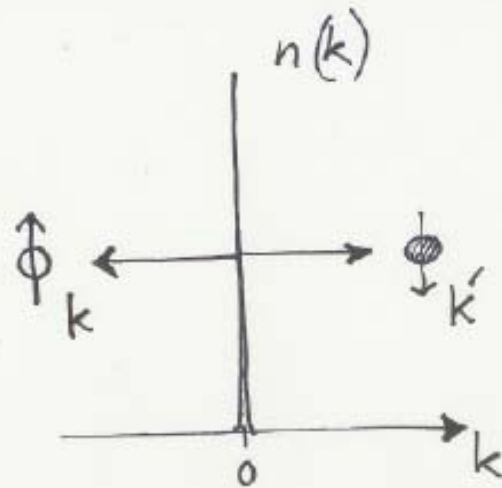
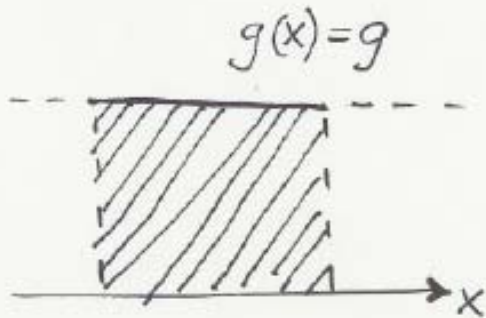


We have calculated Glauber's second order correlation function in momentum space:

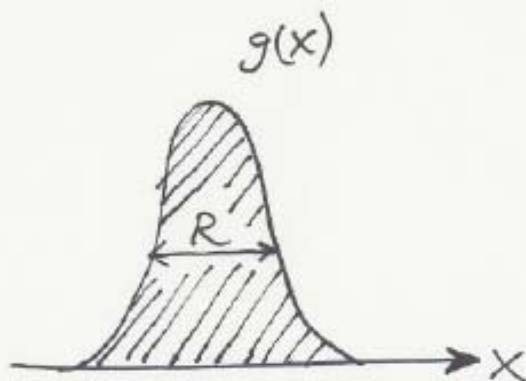
$$g_{ij}^{(2)}(k, k', t) = \frac{\langle \hat{a}_i^\dagger(k, t) \hat{a}_j^\dagger(k', t) \hat{a}_j(k', t) \hat{a}_i(k, t) \rangle}{n_i(k, t) n_j(k', t)}$$

Inhomogeneity gives a width for the correlations

Uniform MBEC

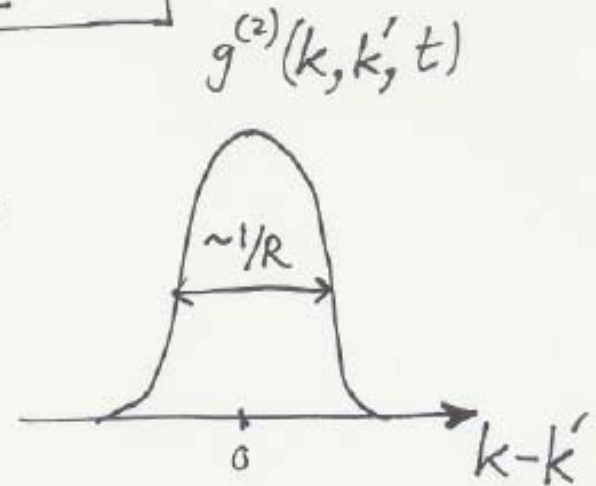
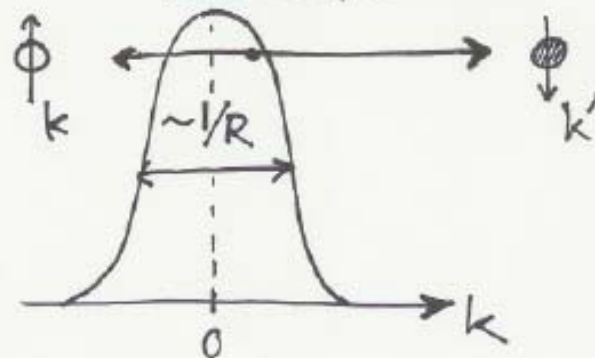


Non-uniform



$k' = -k$ in center of mass frame

Lab. frame:



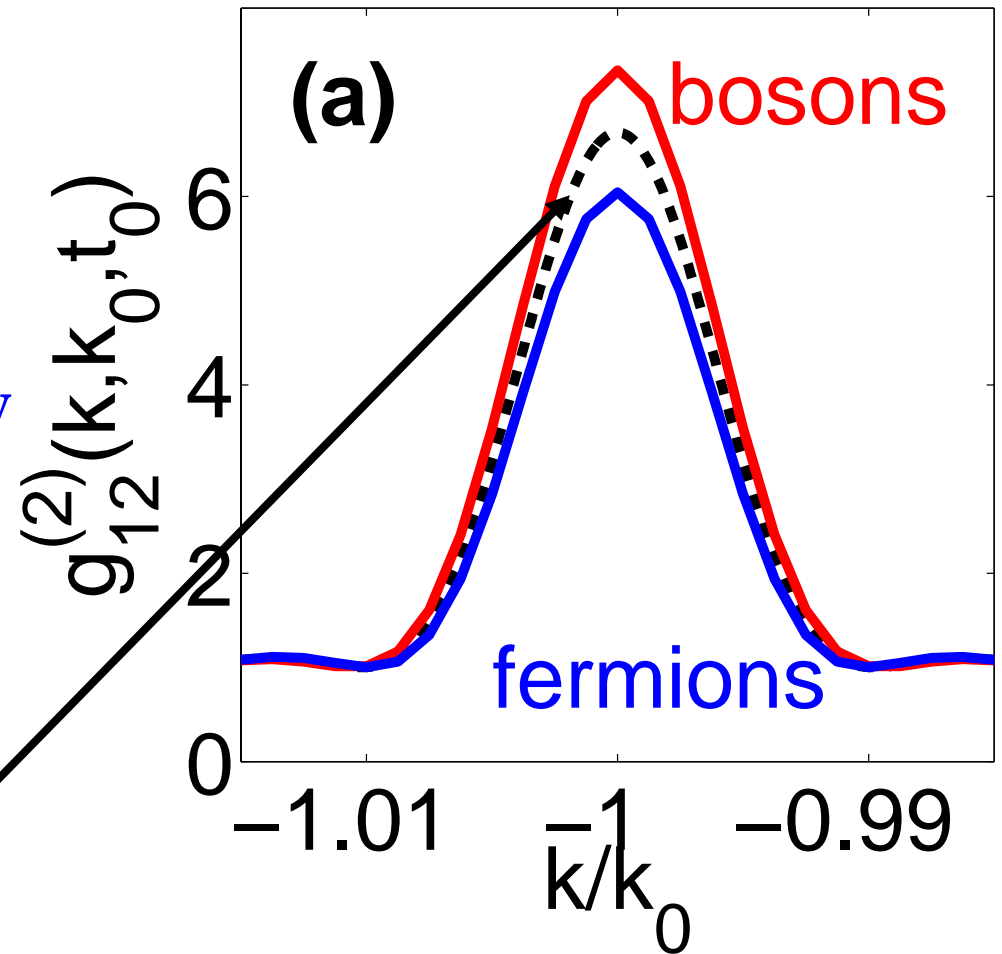
Back-to-back (BB) correlations $g_{1,2}^{(2)}(k, k', t)$

(a) Back-to-back correlations due to momentum conservation

We have derived an analytical asymptotes (dashed line), strictly valid in the short time limit.

But useful even for $t/t_0 \sim 1$.

Solid lines are from a numerical calculation.



$$g_{12}^{(2)}(k, k', t) \simeq 1 + \frac{9\pi^2}{16t^2\chi^2\rho_0} \frac{(J_1[(k+k')R_{TF}])^2}{[(k+k')R_{TF}]^2}$$

Observations from the field of particle physics:

Collinear correlation (CL) $g_{j,j}^{(2)}(\mathbf{k},\mathbf{k}',t)$, $j=1,2$ (same spin state)

Bosons

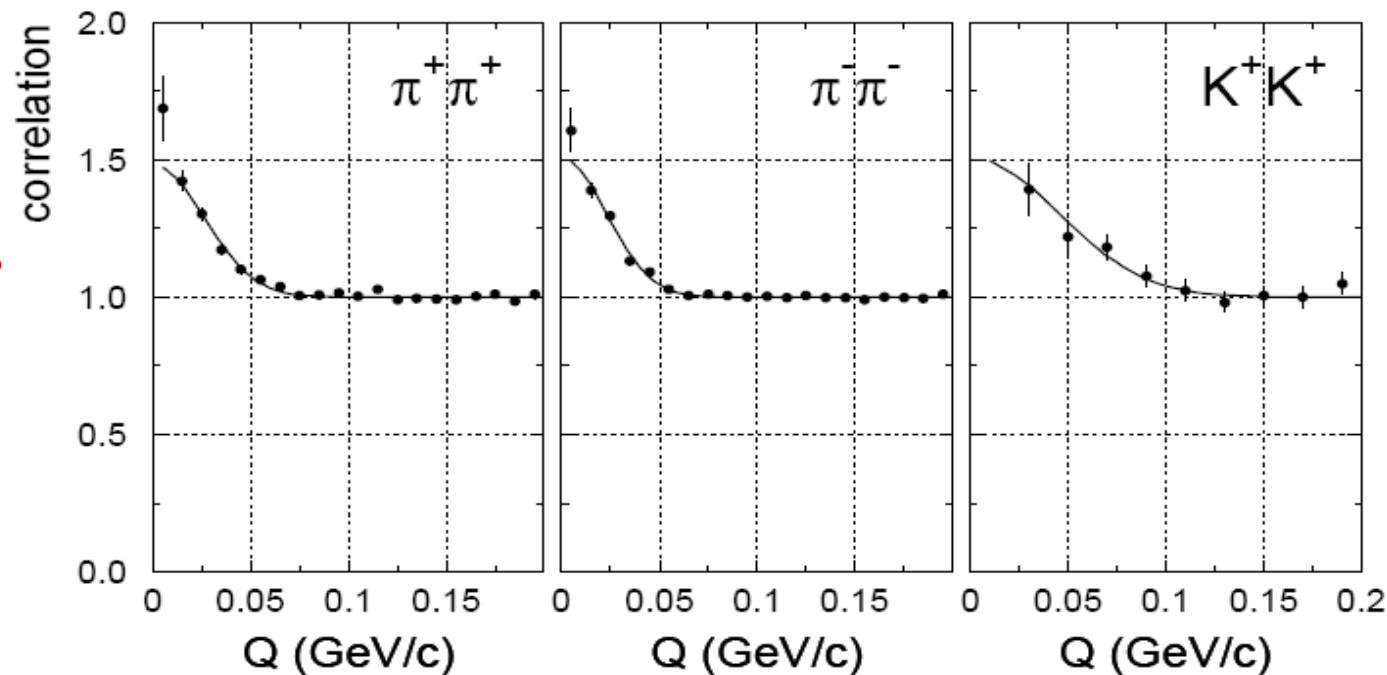
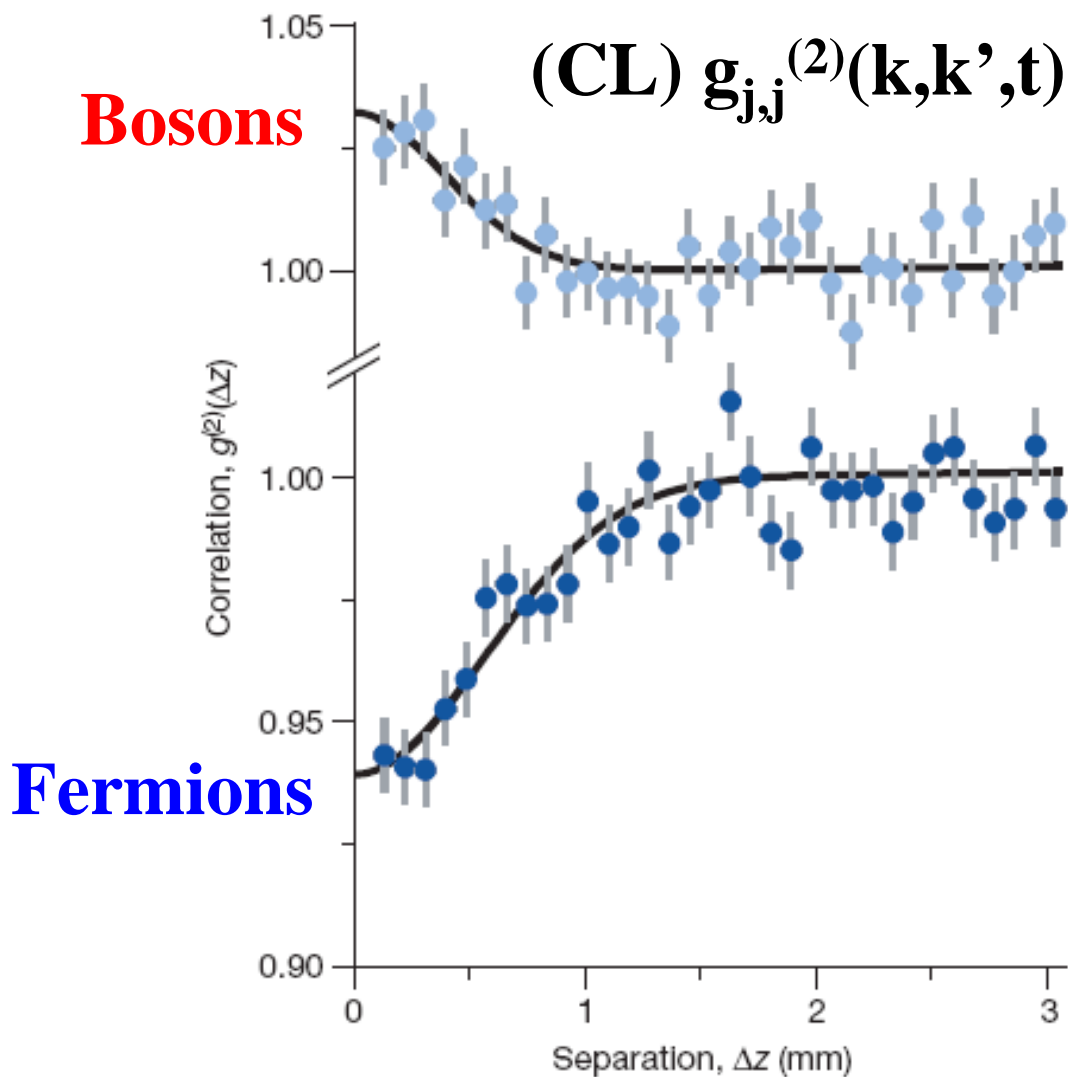


Fig. 2. Two-particle correlation function for $\pi^+\pi^+$, $\pi^-\pi^-$, and K^+K^+ pairs in collisions of Au on Au at 10.8 GeV/A [2].

From: D. Miskowiec *et al.*, E877 collaboration, Nucl. Phys. A610, 237c (1996).
In: G. Baym, Acta Physica Polonica (arXiv:nucl-th/9804026)

Observations from the field of ultra-cold atoms:



**T. Jelte *et al.*, Nature
445 (2007) 402.**

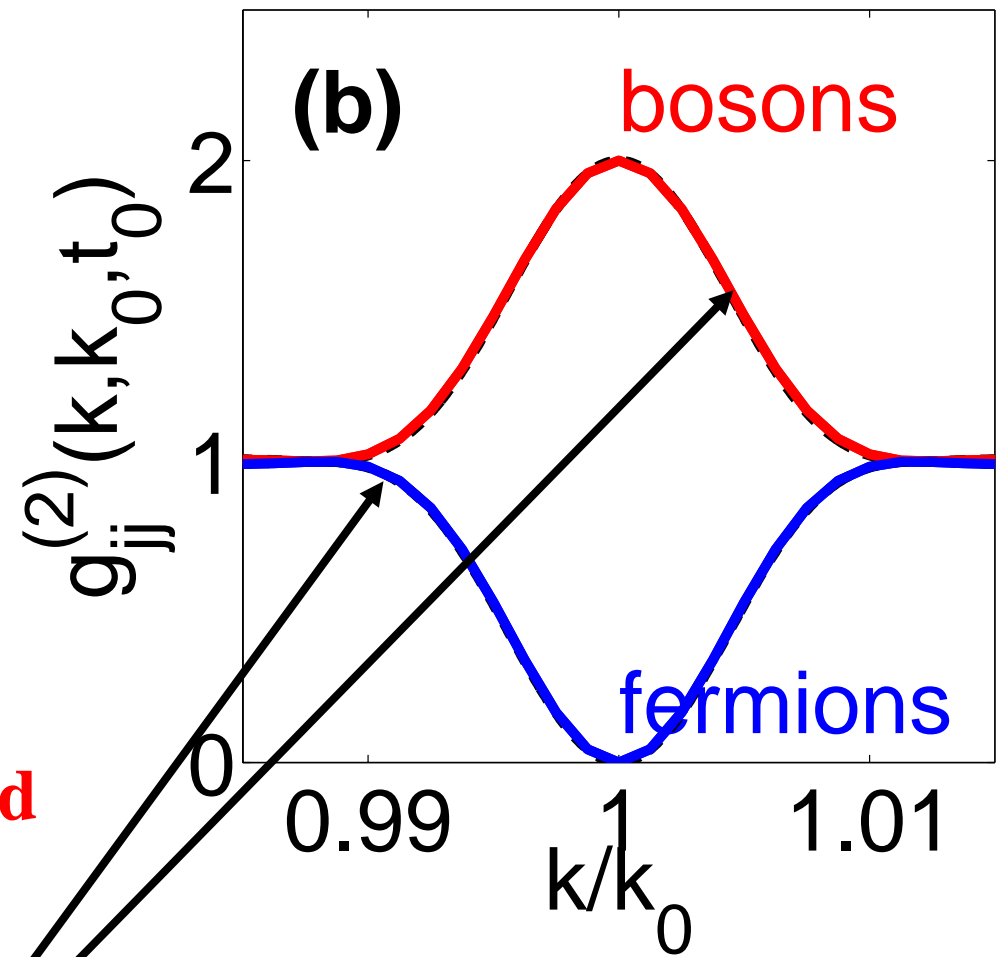
**See also: M. Henny
et al., Science 284,
296 (1999). For
'anti-bunching of
electrons' in a solid
state device.**

Figure 2 | Normalized correlation functions for $^4\text{He}^*$ (bosons) in the upper plot, and $^3\text{He}^*$ (fermions) in the lower plot. Both functions are measured at the same cloud temperature ($0.5 \mu\text{K}$), and with identical trap parameters.

Collinear (CL) correlations, molecular dissociation

(b) Collinear correlations due to particle statistics, (like Hanbury Brown and Twiss for photons).

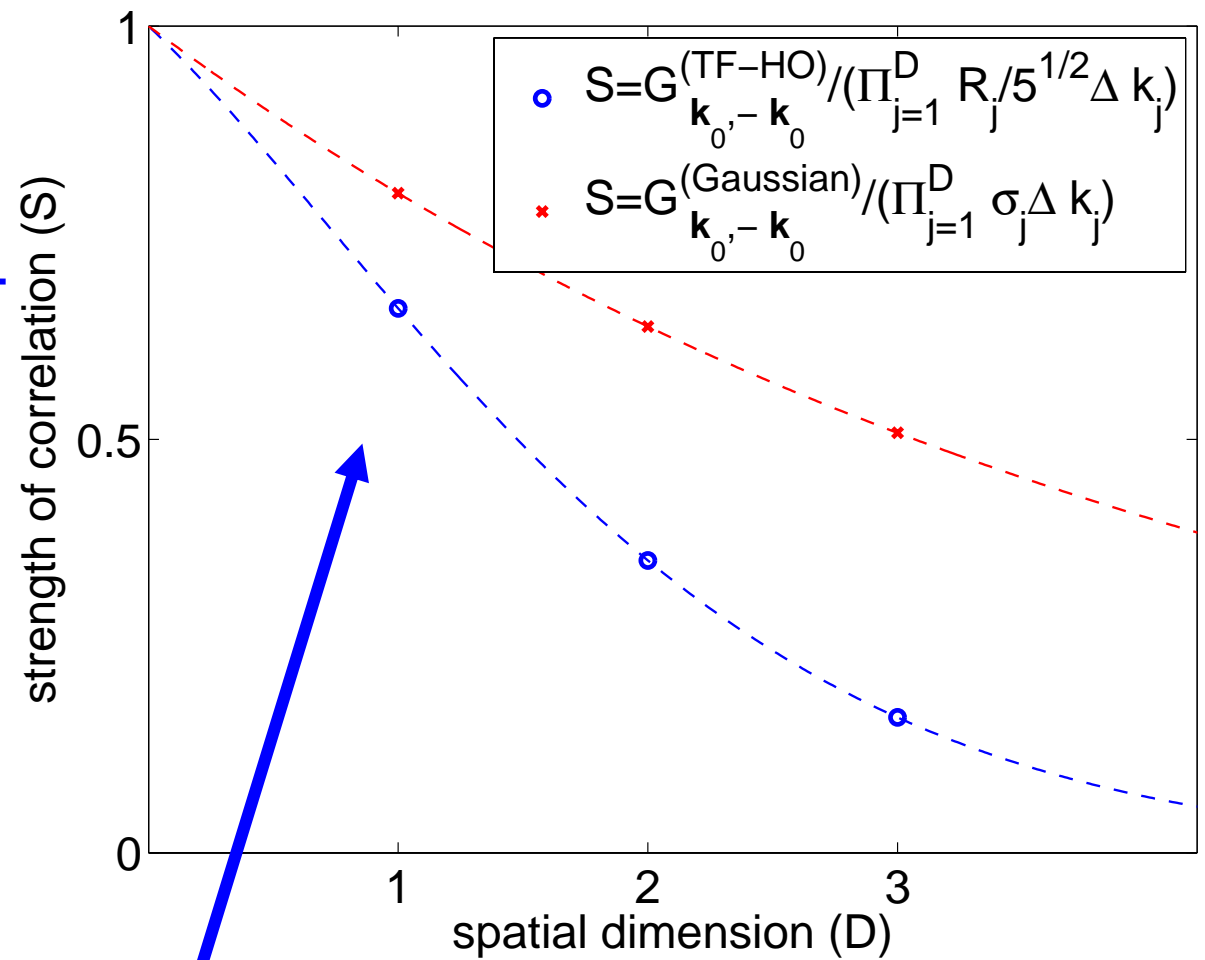
We have derived an analytical asymptote (dashed lines), strictly valid in the short time limit. But useful even for $t/t_0 \sim 1$ as here. **Solid lines** are from a numerical calculation.



$$g_{jj}^{(2)}(k, k', t) \simeq 1 \pm \frac{9\pi}{2} \frac{\left(J_{3/2} \left[(k - k') R_{\text{TF}} \right] \right)^2}{\left[(k - k') R_{\text{TF}} \right]^3}$$

Where is the low-dimensional physics?

*8:th Nordforsk Nordic
network meeting on
Low-dimensional physics*



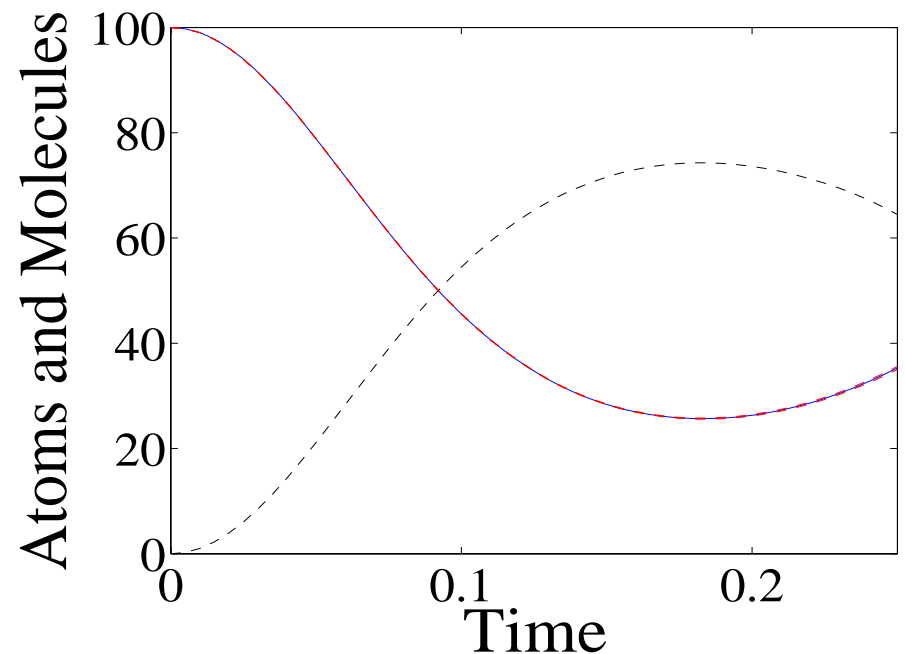
Strongest correlation in low dimension!

Future project: Fermionic phase-space representation

$$\hat{H} = \hat{H}_0 - i\hbar\chi \int d\mathbf{x} \left(\hat{\Psi}_0^\dagger \hat{\Psi}_2 \hat{\Psi}_1 - \hat{\Psi}_1^\dagger \hat{\Psi}_2^\dagger \hat{\Psi}_0 \right)$$

Treat the full Hamiltonian, using the Gaussian operator method introduced by Corney and Drummond [PRL 93 (2004)], this leads to stochastic c-number differential equations.

First principle phase space calculation for 100 molecules and 100 fermionic momentum modes:



Summary of our results for non-uniform condensates

- * **Minor differences in density and total atom numbers for non-uniform and size-matched uniform system.**
- * **For correlations it is crucial to treat the inhomogeneity.**
- * **Correlation functions expressed analytically for small times.**
- * **First principle calculations under development in order to investigate the effect of molecular depletion and atom-atom interaction.**

Thank you for your attention!