QFT-I problem set 3

(Fawad Hassan)

Deadline: Wednesday, Feb 01, 2012

1. (a) Starting from the Yang-Mills Lagrangian for the $SU(2)_W \times U(1)_Y$ gauge fields W_i^μ and B^μ, work out the Lagrangian for the physical fields $A^\mu, Z^\mu, W^\mu, W^{\dagger\mu}$, including the interaction terms.

   (b) Starting from $D_\mu\Phi^\dagger D^\mu\Phi$, where Φ is the Higgs doublet in the electroweak theory, show that after spontaneous symmetry breaking, the gauge fields $W, W^\dagger, Z$ become massive while the photon field A remains massless.

   (c) Derive the Feynman rules for the vertices of the type $WW^\dagger AA$, $WW^\dagger A$ and $WW^\dagger AZ$.

2. Consider the elastic electron-neutrino scattering processes, $\nu_\mu + e^- \to \nu_\mu + e^-$ and $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ in electroweak theory. Write down the expressions for the Feynmann amplitudes.

3. Starting from the generalized Higgs-neutrino coupling term (see Mandl and Shaw for details on the conventions),

$$-G_{l'l}\bar{\Psi}_{l'}^L(x)\psi_{\nu_l}^R(x)\tilde{\Phi}(x) - G_{l'l}^*\tilde{\Phi}^\dagger(x)\bar{\psi}_{\nu_l}^R(x)\Psi_{l'}^L(x) \qquad (A)$$

   show that in the unitary gauge, it reduces to

$$-\frac{1}{\sqrt{2}}\sum_j \lambda_j\bar{\psi}_j(x)\psi_j(x)[v+\sigma(x)]$$

   where $\psi_j = \sum_l U_{jl}\psi_{\nu_l}$ and U is the unitary matrix that diagonalizes the Hermitian coupling matrix G, *i.e.*, $(UGU^\dagger)_{ij} = \lambda_i\delta_{ij}$. Hence, show that the coupling term (A) leads to eigenstate neutrinos ν_j associated with the fields $\psi_j(x)$ with masses

$$m_j = \lambda_j v/\sqrt{2}$$

   Draw the Higgs-neutrino (ν_j) interaction vertex and show that it comes with vertex factor $(-i/v)m_j$.

4. The Lagrangian for a massive vector field W_μ,

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m_W^2 W_\mu W^\mu\,, \qquad F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$

   is not invariant under $U(1)$ gauge transformations, hence a Lorenz gauge cannot be imposed. Show that the condition $\partial_\mu W^\mu = 0$ is, however, implied by the equation of motion. How many degrees of freedom will W_μ have?

5. Problem 19.2 (page 448) from Mandl and Shaw (II^{nd} edition).