An Exact Solution for the Half-filled Lowest Landau Level

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“The story of the half-filled Landau level on a torus as a function of its circumference”

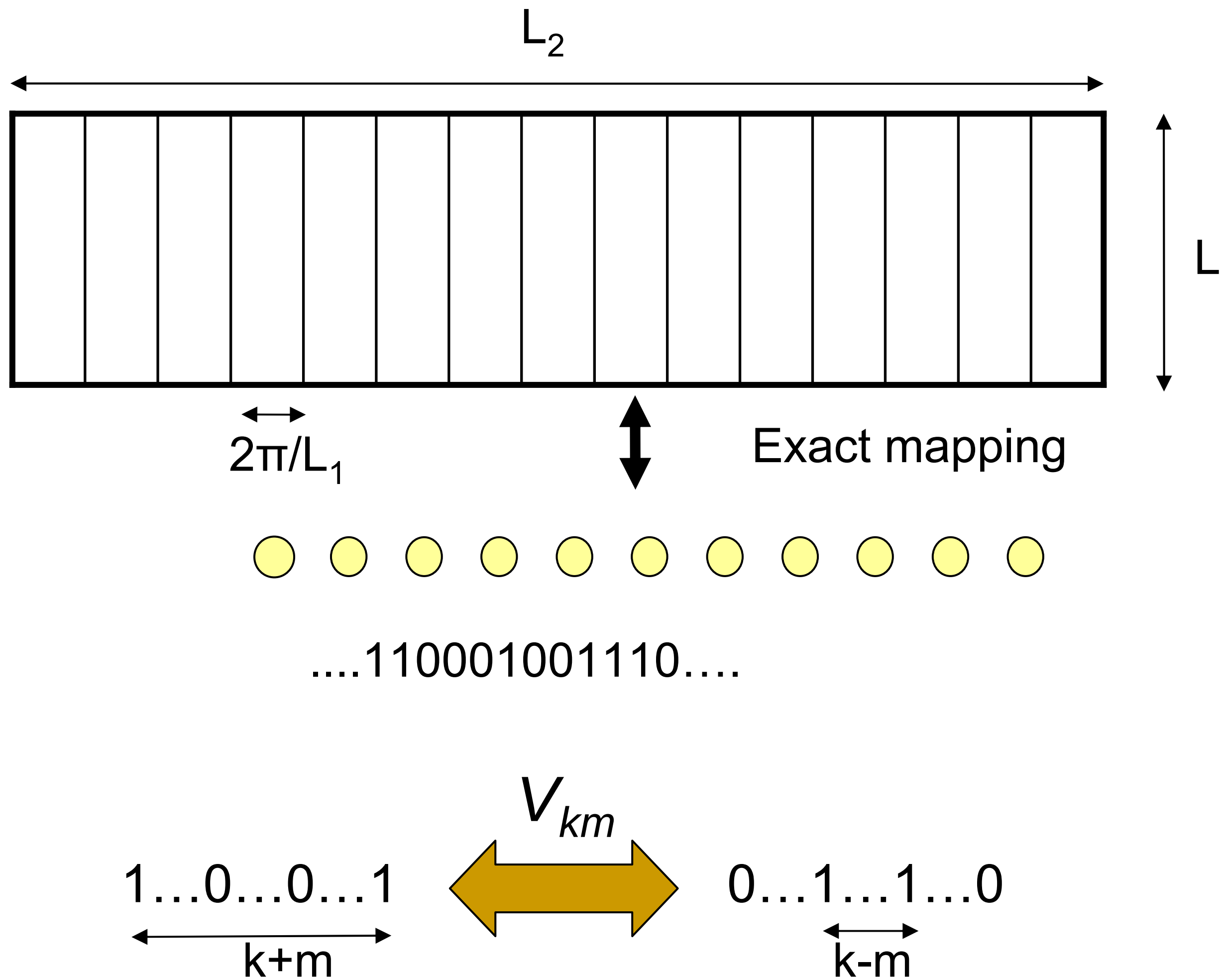
The lattice model

2D electron gas

1D spin-chain

Many-particle states

Two-body interaction



The thin torus ($L_1 \to 0$)

- Hopping is exponentially damped---all energy eigenstates have fixed charges.
- Two ground states 101010… and 010101…
- These states have a gap to excitations---the lattice is rigid and there are no phonons.

L_1 increases…

- As the torus grows hopping becomes important and starts to compete with the electrostatic terms.

- However, 101010… is not very ‘hoppable’ and there is a phase transition to a qualitatively different phase. For Coulomb interaction this happens at $L_1 \sim 5$ (magnetic lengths).

A solvable model

- On a thin, but finite torus it is a good approximation to truncate the Hamiltonian, keeping the following terms:

V_{10} : 11

V_{20} : 101

V_{21}

1001 $\Longleftrightarrow$ 0110

$$H_{solv} = \sum_k V_{10}\, n_k n_{k+1} + V_{20}\, n_k n_{k+2} - V_{21}(c_k^+ c_{k+1} c_{k+2} c_{k+3}^+ + H.c.)$$

Exact solution

- We define a subspace $\mathcal{H}'$ by demanding that each pair of nearby sites (2p-1,2p) contains exactly one electron.
- $\mathcal{H}'$ contain the low-energy sector and is preserved by H_{solv}.
- The states in $\mathcal{H}'$ are labeled by ↑=10 and ↓=01 and the dipole operators $s_p^+ = c_{2p}^+ c_{2p-1}$ and $s_p^- = c_{2p}^+ c_{2p-1}$ describe hard-core bosons in $\mathcal{H}'$.
- At the point $V_{10}=2V_{20}$ the Hamiltonian is simply

$$H_{solv} = V_{21} \sum_n (s_n^+ s_{n+1}^- + h.c.)$$ (+ constant term)

Spin 1/2 XY model!

The bosons can be expressed in terms of fermions by a Jordan-Wigner transformation

$$s_p^- = e^{i\pi \sum_{j=1}^{p-1} d_j^+ d_j} d_p$$

→ Free fermions!

Fourier transformation →
$$H_{solv} = 2V_{21} \sum_k \cos k \, \tilde{d}_k^+ \tilde{d}_k$$

The ground state is a filled 1D Fermi-sea of neutral dipoles with gapless excitations.

Luttinger liquid description

Perturbation theory….

$$\Delta H = \sum_{n,k} \alpha_k (s_n^+ s_{n+k}^- + h.c.) + \lambda_k s_n^z s_{n+k}^z$$

(+ quartic and higher order terms)

RG: no phase transition near the solvable point.

→ Luttinger liquid of neutral dipoles

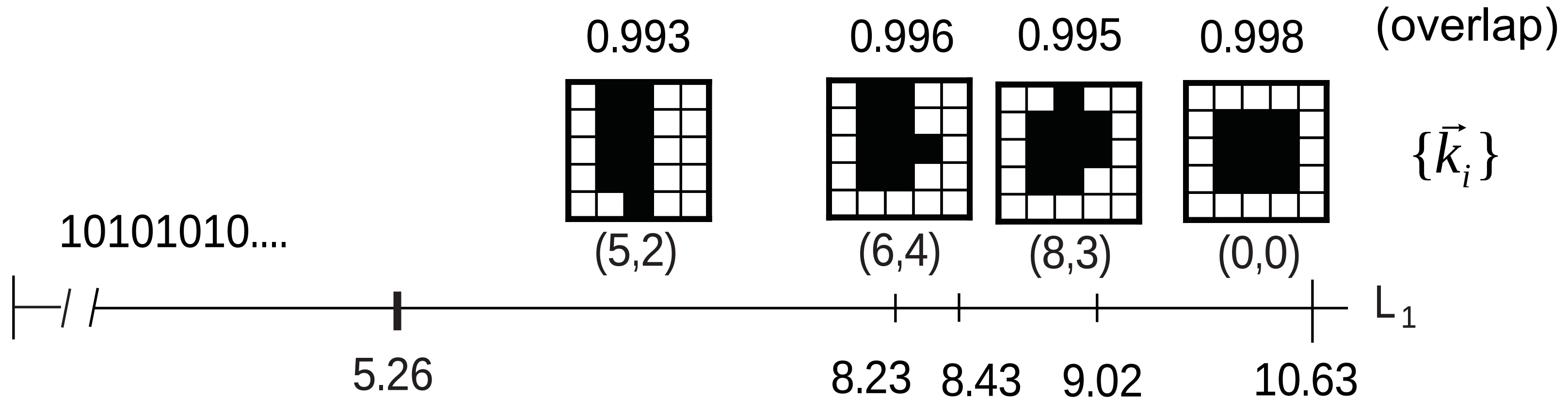
Confirmed by numerics! (DMRG and exact diagonalization)

Conclusion: The exact solution describes the low-energy sector for a finite range of L_1.

Connection to the bulk

- The exact solution corresponds to the 'Rezayi-Read state', for a special choice of parameters $\{\vec{k}_i\}$.

$$\Psi_{RR} = Det_{ij}(e^{i\vec{k}_i \cdot \vec{R}_j})\Psi_{1/2}$$



- Checked in exact digonalization for up to ten electrons.

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Conclusions

- There are two different phases of the half-filled LLL---the gapped phase at very small L_1 and the gapless phase at $L_1 > 5$.
- The exact solution describes the physics on the thin torus---and more importantly---qualitatively also of the bulk system.
- Possible Luttinger liquid effects in the bulk?