

Tentamen i Kosmologi och astropartikelfysik FK7007
9.00-14.00, 2011-06-01

Hjälpmedel: bifogad formelsamling, miniräknare och *Physics Handbook*.

1. a) Starting from the Einstein eqn., explain why the cosmological constant, Λ , is associated with the vacuum energy density. (2p)
b) Describe the observational evidence for *dark energy*. Why is the measured value of Λ “unnatural”? (2p)
2. a) Describe the observational evidence for the Big Bang model (2p)
b) Starting from the Friedmann eqn explain the relation between the future (asymptotic) expansion of the universe and curvature for $\Lambda = 0$. (2p)
3. a) Show that in a flat Universe $\dot{H} = -4\pi G(\rho + p)$ (2p)
b) How does the expression above depend on redshift for i) radiation, ii) matter and iii) Λ dominated epochs? (2p)
4. a) Explain what is the “horizon problem”. How does the proposed Inflationary mechanism solve it? (1p)
b) For a flat universe, the angular diameter distance is given by

$$d_A = \frac{c}{(1+z_s)} \int_0^{z_s} \frac{dz}{H(z)}.$$

- Calculate d_A to a surface of last scattering, $z_s \approx 10^3$ in two types of universes: i) flat $\Omega_M = 1$ and ii) flat $\Omega_\Lambda = 1$. (2p)
c) What are the corresponding values for the luminosity distance to the same redshift? (1p)

5. a) Show that the product aT is invariant for radiation in an expanding universe. (1p)
b) Can you think of any processes that could change that invariance in the present universe? (1p)
c) Show that a radiation dominated universe cools down as $T \propto t^{-1/2}$. (1p)
d) Explain why neutrinos decouple from matter much earlier than photons. (1p)

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6. The number density of non-relativistic particles in thermal equilibrium is given by

$$n_{NoRe} = g_i \left(\frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{m}{T}}.$$

The interactions which keep neutrons and protons in thermal equilibrium freeze out at roughly $T=0.8$ MeV.

- a) Estimate the abundance of ${}^4\text{He}$ given that all neutrons end up in ${}^4\text{He}$. (3p)

- b) Why is this value higher than the measured 24%? (1p)

$$(m_p c^2 = 938.28 \text{ MeV}; m_n c^2 = 939.57 \text{ MeV})$$

Good luck!

Compilation of useful formulas

Equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu};$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3\frac{p}{c^2} \right) + \frac{\Lambda}{3};$$

$$\frac{p}{c^2} = w \cdot \rho;$$

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) = 0$$

$$q_0 = - \left(\frac{\ddot{a}(t_0)}{a(t_0)} \right) \frac{1}{H_0^2}$$

Constants

Newton's constant	G	$6.672 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Speed of light	c	$2.998 \times 10^8 \text{ m s}^{-1}$
Planck's constant	$\hbar = h/2\pi$	or $3.076 \times 10^{-7} \text{ Mpc year}^{-1}$
Boltzmann's constant	k_B	$1.055 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$ $1.381 \times 10^{-23} \text{ J K}^{-1}$
Radiation constant	$\alpha = \pi^2 k_B^4 / 15 \hbar^3 c^3$	or $8.619 \times 10^{-5} \text{ eV K}^{-1}$
Electron mass	$m_e c^2$	$7.565 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$ 0.511 MeV
Proton mass	$m_p c^2$	938.3 MeV
Neutron mass	$m_n c^2$	939.6 MeV
Planck mass	$M_{Pl} c^2$	$1.2 \cdot 10^{19} \text{ GeV}$
Thomson cross-section	σ_e	$6.652 \times 10^{-29} \text{ m}^2$
Neutron halftime (free neutron)	$t_{n,1/2}$	614 s
Hubble constant	H_0	$100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$
h		or $H_0^{-1} = 9.77 \text{ h}^{-1} \times 10^9 \text{ years}$ 0.74 ± 0.05

Conversion factors

$$\underline{1 \text{ pc} = 3.261 \text{ light-year} = 3.086 \times 10^{16} \text{ m}}$$

$$1 \text{ year} = 3.156 \times 10^7 \text{ s}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$\underline{1 M_\odot = 1.989 \times 10^{30} \text{ kg}}$$