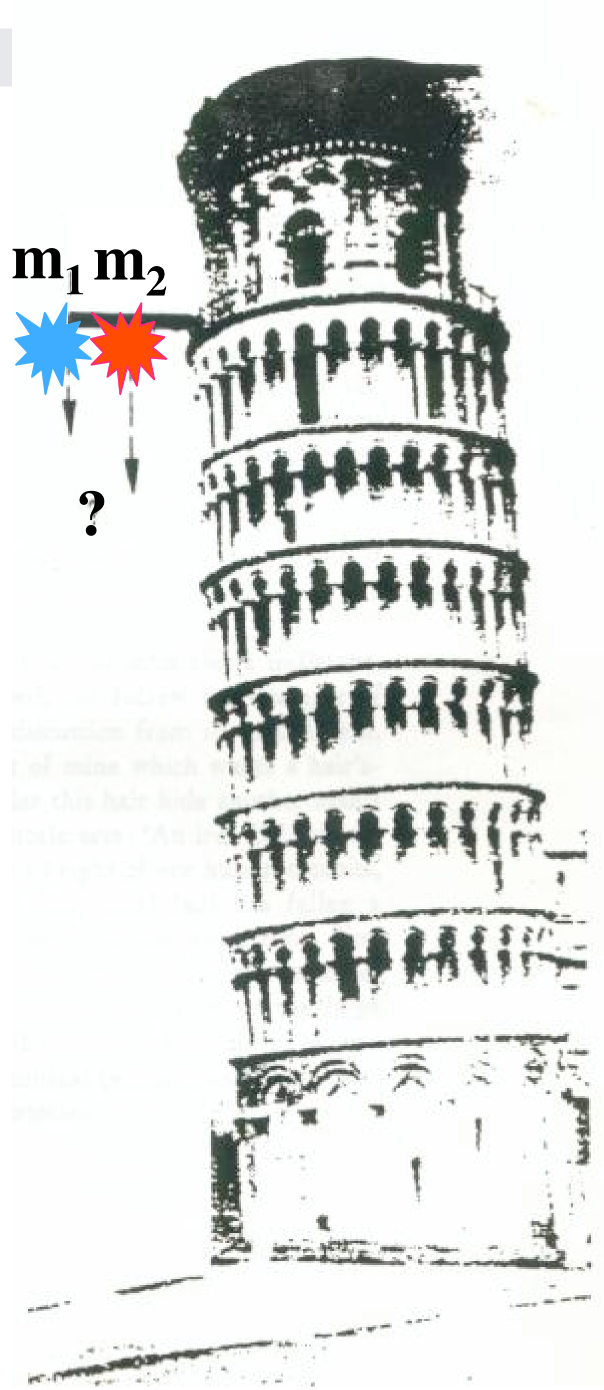
Precisionsmassbestämning - med en atom i vågskålen eller Hur man väger atomer och varför?

Från vikten av ett kilogram till massan av en neutrino

Reinhold Schuch, FYSIKUM, Stockholm Universitet

En historia om(av) vikt och(om) massa

För mätning av massa användar man (ofta omedvetet) materiens tunga massa (gravitation) och bestämmer mängd, dvs den tröga massan



gravitationskraft = trög massa x acceleration

Newtons andra lag: $G\, Q_E Q_i / R^2_E = m_i g$

EP: $Q_i/m_i = g\,(R^2/Q_E\, G)$ **=konstant, oberoende av i**

Ekvivalens-Principen: Ekvivalens mellan tung och trög massa

En historia om vikt och massa

1 kg ⇔ 1Ltr vatten vid 3,98grad...

Ur-Kilogram" i Labor des "Bureau International des Poids et Mesures" i Sèvres vid Paris

Kilogram enda dim. i SI måttsystem som inte har någon atomistisk definition.

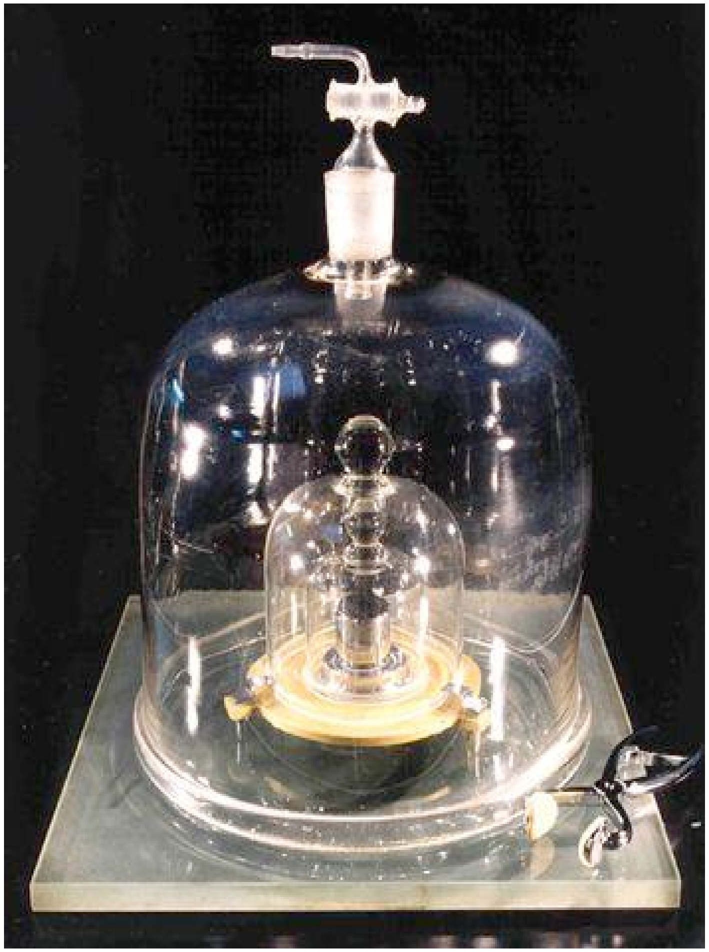
James Clerk Maxwell var först med förslag....

Egentligen finns def.: en Mol 12C har massan 12 g.

Men fortfarande inte användbar. Dock om man kunde räkna antal (N_A=6,022.. · 10^{23}) C-atomer noggrant...

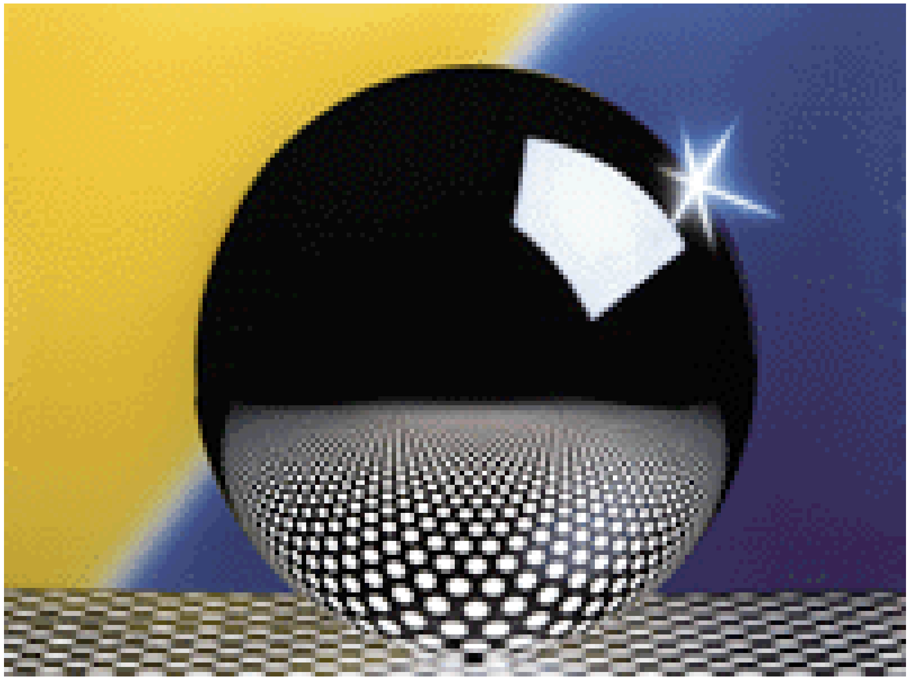
Vad väger ett kilo?

Idag



Kilogram prototype
Bureau International des Poids et Mesures

Framtiden?



Boll gjord av en enda silikonkristall

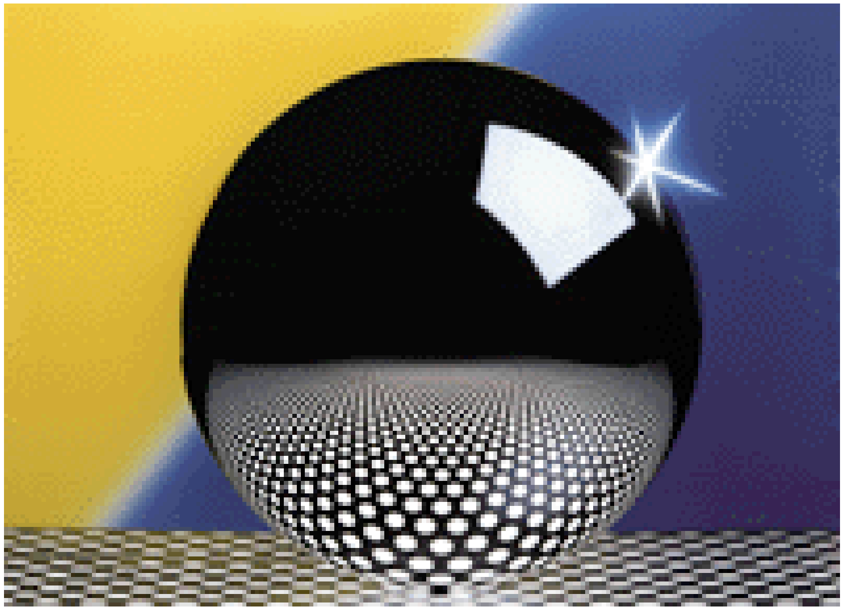
Krav: $\delta m/m \leq 1\ 10^{-8}$

The Avogadro Project:

"The kilogram is the mass of *N* ^{28}Si-atoms."

"Recept"

- **Tillverka en perfekt Si kristall** ✓
- **Gör en boll av den** ✓
- **Mät diametern** ✓ $\leq$ 1nm
- **Bestäm gitterparametern**
- **Mät renheten**
- **Beräkna antalet Si atomer**
- **Mät masskvoten ^{28}Si/^{12}C**

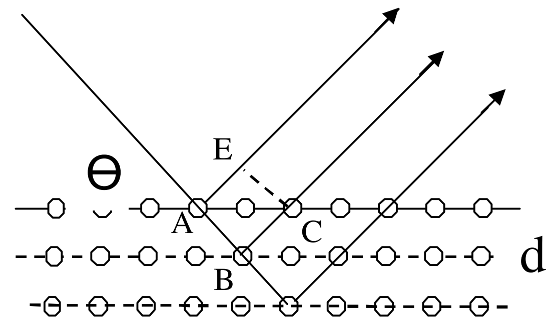


Allt måste göras med en osäkerhet av 10^{-9} eller bättre!

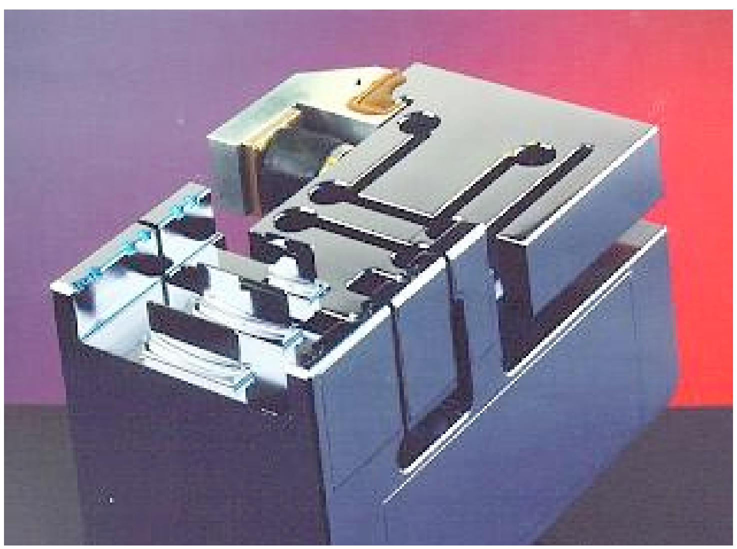
Bestämma lattice parametern:

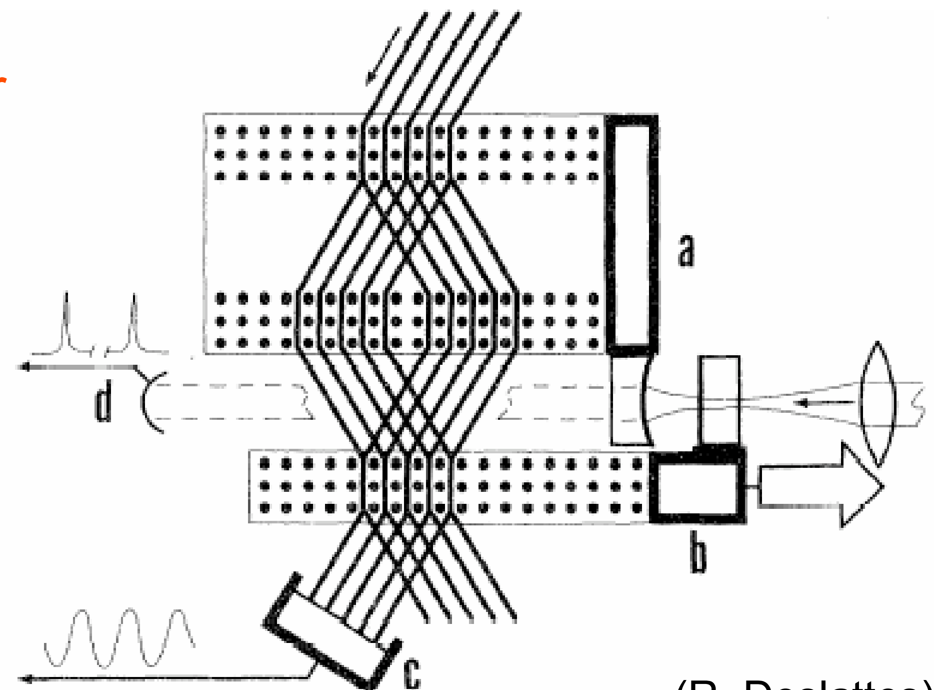
Röntgendiffraktion med Si kristall
Braggvillkor
Intensitetsmax. för ϴ

$$n\lambda = 2d\sin\Theta$$



•Braggvillkoret kopplar d och λ med Braggvinkeln ϴ.
•Röntgen – Laser-interferometer kopplar λ med meterenheten: Δ = 6057.80211Å gul 86Kr linje.
Mätosäkerhet 12 Attometer (10^{-18} m)





(R. Deslattes)

FIG. 1. Principle of simultaneous x-ray and optic[al] interferometry of a common base line. Stationary a[nd] moving parts are labeled a and b, respectively. Th[e]

The Avogadro Project:

“The kilogram is the mass of N ^{28}Si-atoms.”

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Tillverka en perfekt Si kristall ✓

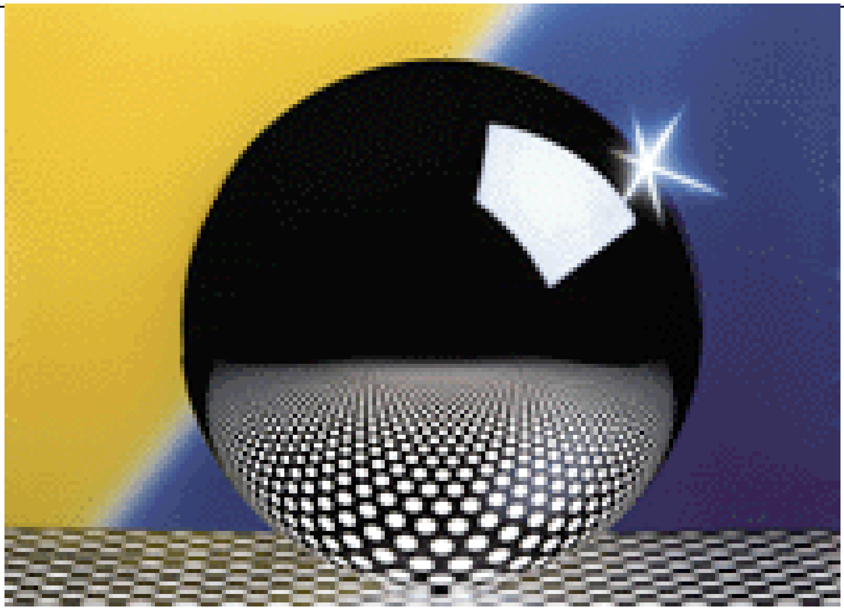
Gör en boll av den ✓

Mät diametern ✓ $\leq$ 1nm

Bestäm gitterparametern ✓

Mät renheten ✓

Beräkna antalet Si atomer ✓



Mäta masskvoten ^{28}Si/^{12}C **GJORT VID SMILETRAP i Stockholm** ✓

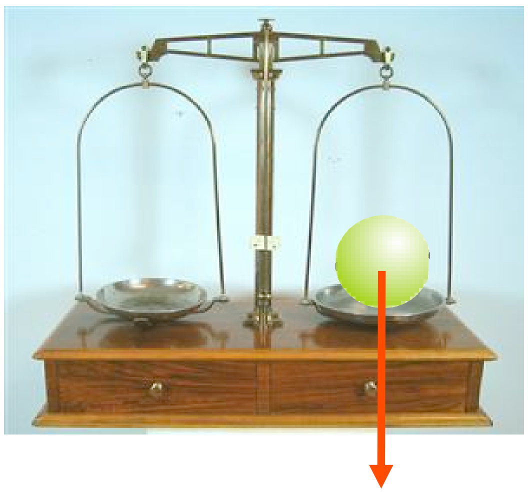
Dock, Kg framställt så här var inte tillräkligt noggrant →

Mät isotopsammansättningen ^{28}Si, ^{29}Si, ^{30}Si och deras massa !!

Allt måste göras med en osäkerhet av 10^{-9} eller bättre!

Hur kan man 'väga' en atom (jon)?

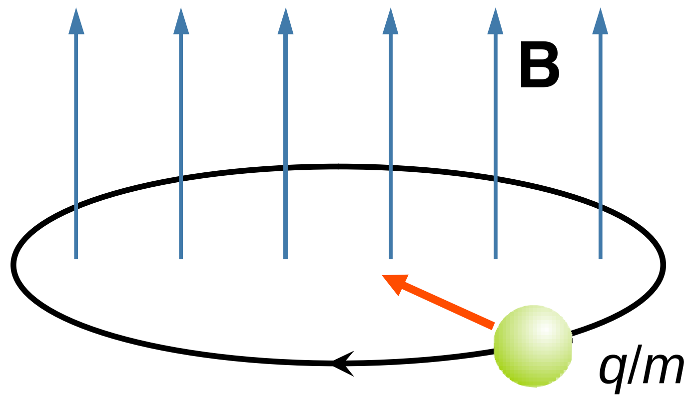
Traditionell massbestämning



Gravitationskraft

$$\overline{F} = m \cdot \overline{g}$$

I en Penningfälla



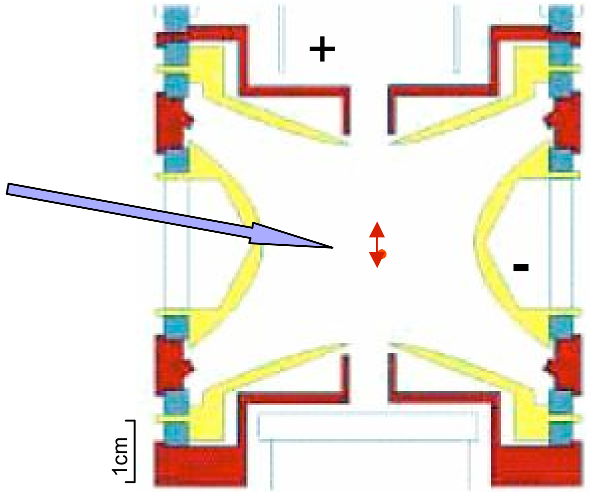
Magnetisk kraft

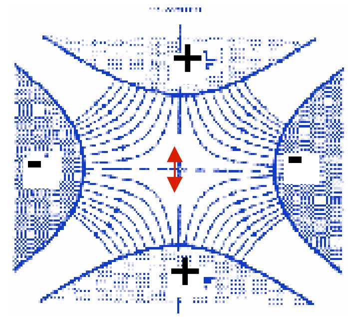
$$\overline{F} = q(\overline{v} \times \overline{B})$$

Cyclotronfrekvens: $$f_c = \frac{1}{2\pi} \cdot \frac{q}{m} \cdot B$$

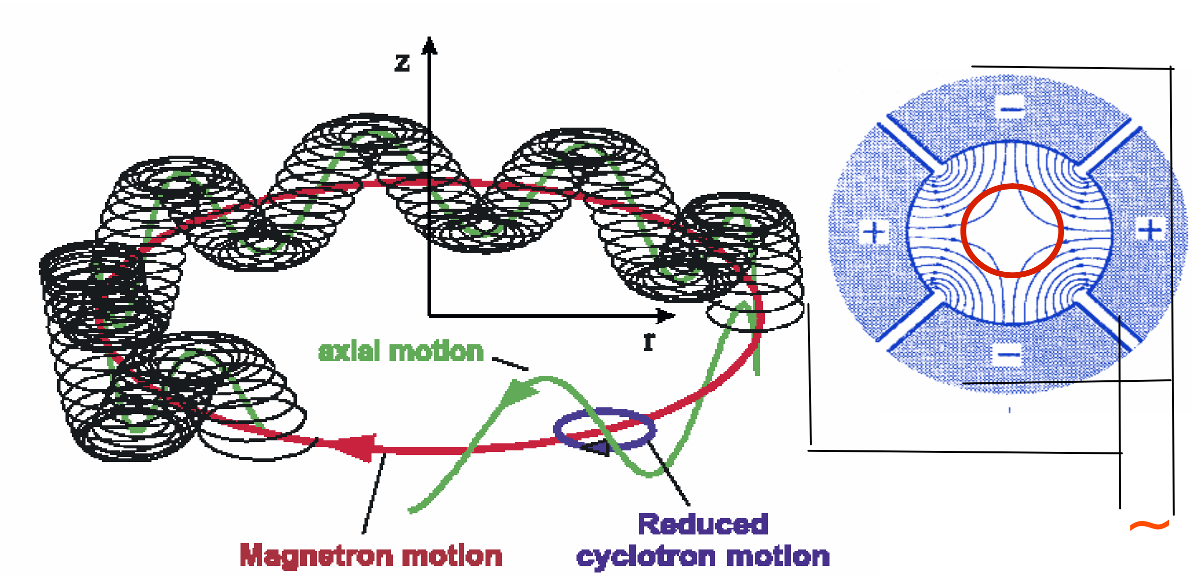
mät f_c i en Penningfälla →

Exitation of an ion in a Penning trap





$$\nu_c = \nu_+ + \nu_- = \frac{1}{2\pi}\frac{qeB}{m}$$



$\nu_- \approx 800$ Hz, $\nu_+ \approx$ 36 MHz, $\nu_z \approx 240$ kHz

Cyclotron Frequency Detection

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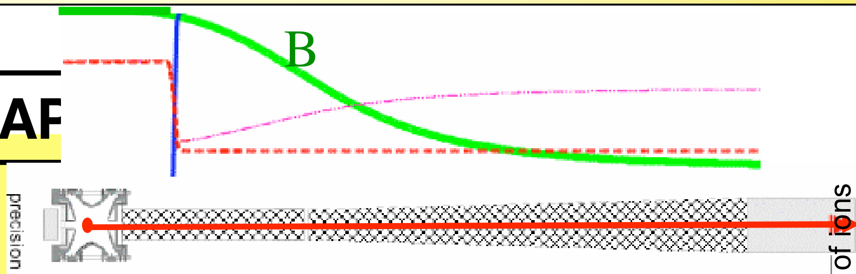
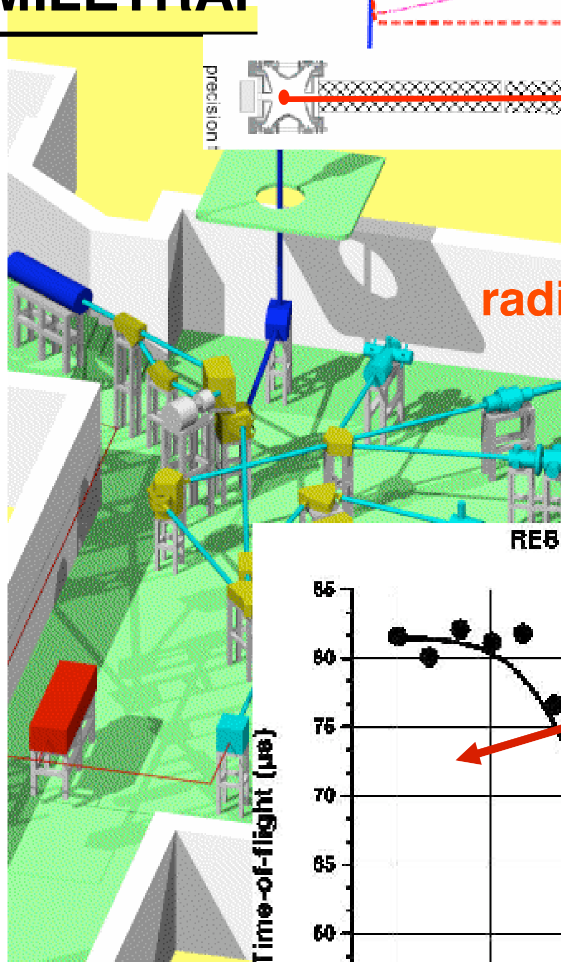
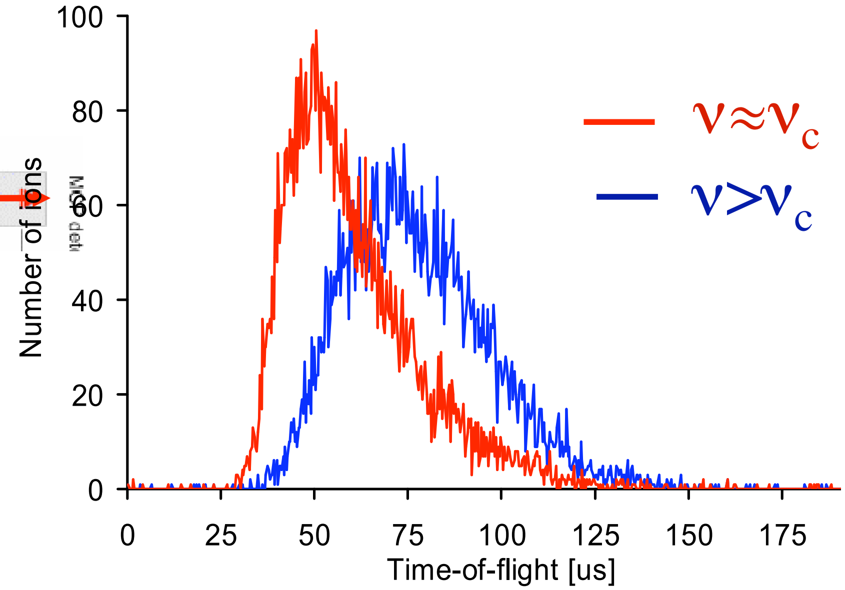
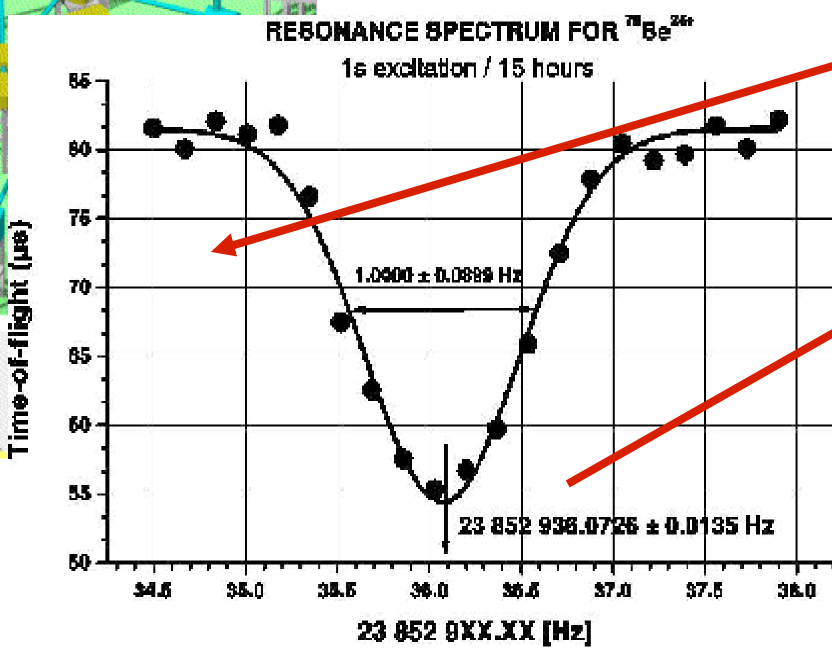




$$\overline{F} = -\mu \cdot \nabla \overline{B}$$

radial ⇒ axial energy



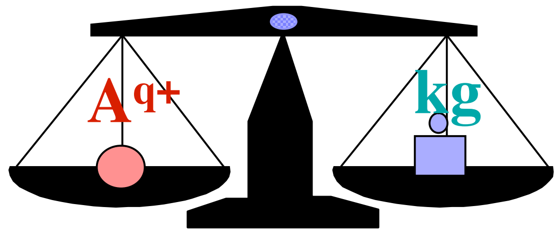


$$R = \frac{\nu_c}{\nu_{cREF}} = \frac{q m_{REF}}{q_{REF} m}$$

$$M_A = \frac{1}{R}\frac{q}{q_{REF}} m_{REF} + q m_{e^-} - E_B(A^{q+})$$

$mc^2 = E_B$

Precisionsmassbestämning



SMILETRAP **S**tockholm **M**ainz **I**on **LE**vitation **TRAP**

- Vi använder högt laddade joner:

precisionen: $\frac{m}{\Delta m} = \frac{\nu_c}{\Delta \nu_c} = \frac{1}{2\pi \Delta \nu_c} \cdot \frac{q}{m} \cdot B$ växer lineärt med ladningen!

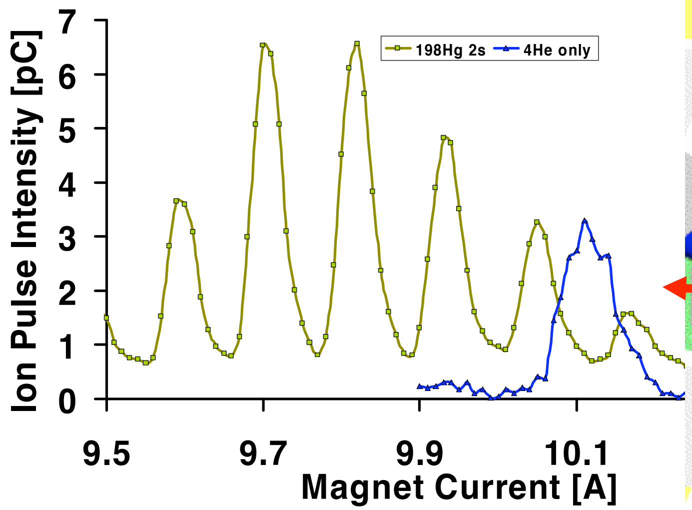
- Vi behöver kalibrera B fältet, med referensmassan ^{12}C

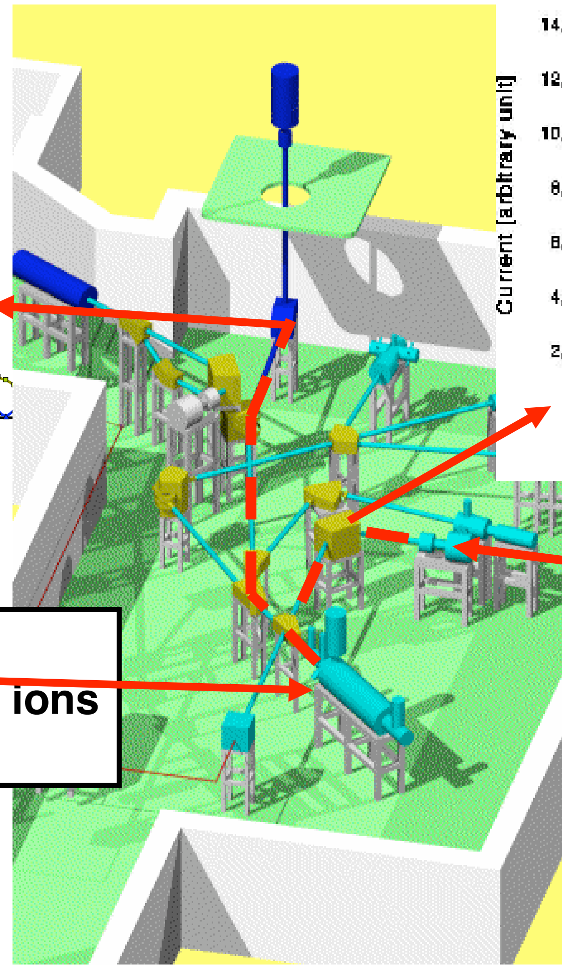
$$R = \frac{\nu_c}{\nu_{cREF}} = \frac{q m_{REF}}{q_{REF} m} \Rightarrow M_A = \frac{1}{R} \frac{q}{q_{REF}} m_{REF} + q m_{e^-} - E_B(A^{q+})$$

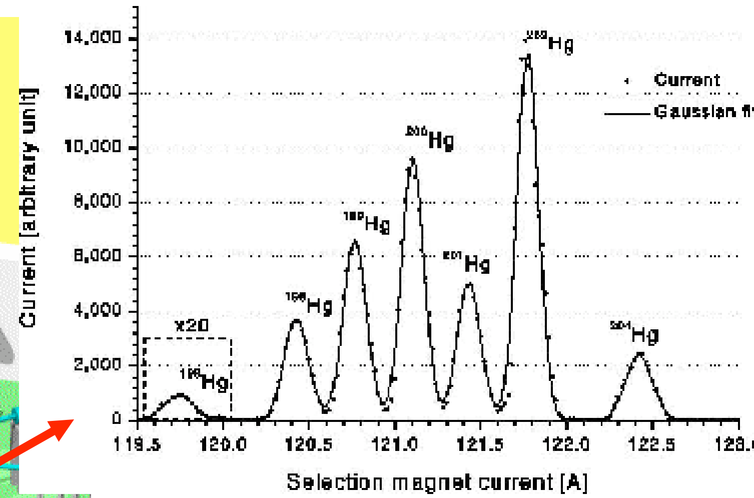
- Använder C^{6+} för att kalibrera Si^{14+}

$\Rightarrow$ nästan samma q/M dvs. nästan samma frekvens!!

Charge-and Isotope-selected Highly Charged Ions :



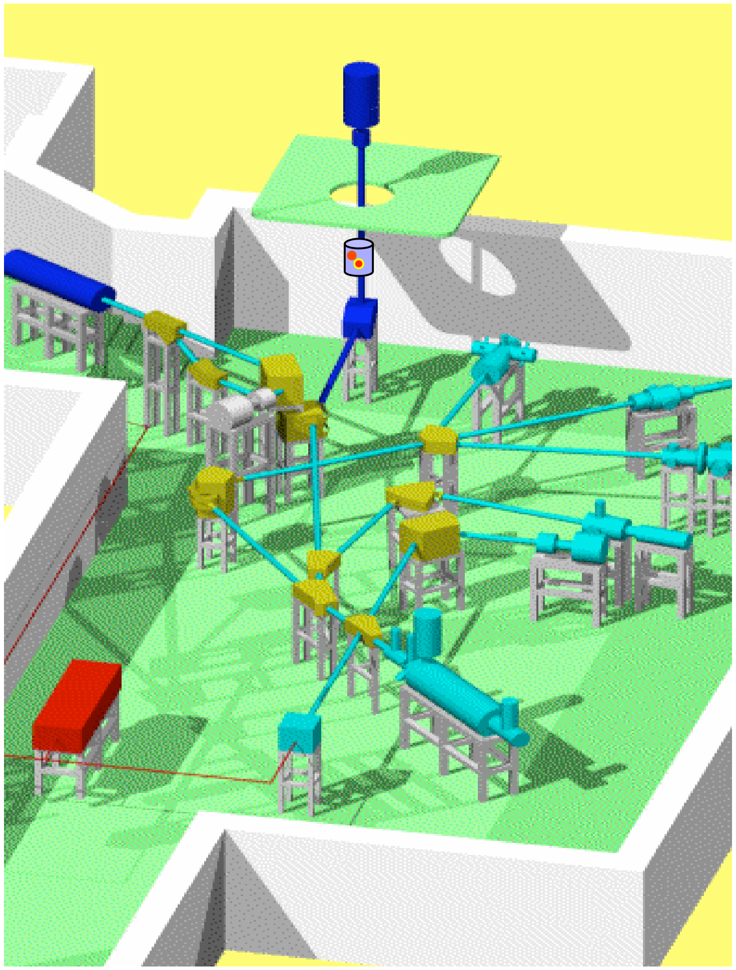




CRYSIS
Production of highly charged ions by electron bombardment

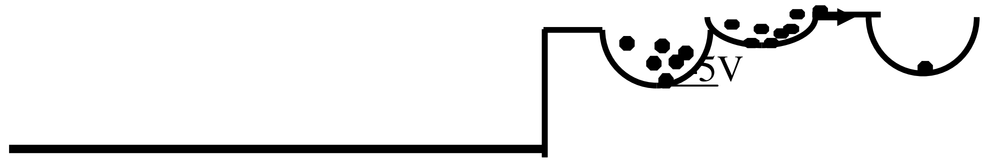
CHORDIS
Production of singly charged isotope separated ions

Ion transport to SMILETRAP and evaporative cooling



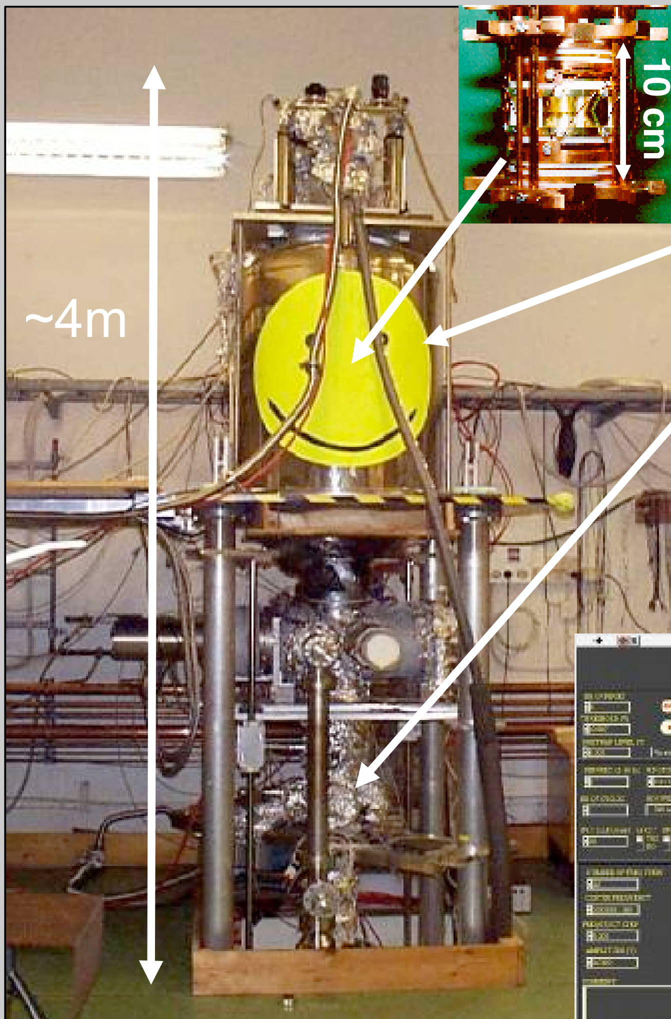
precision trap

evaporative cooling



Finally: 0 or 1 or 2 cold ions in precision trap !

Stockholm-Mainz-Ion LEvitation-TRAP



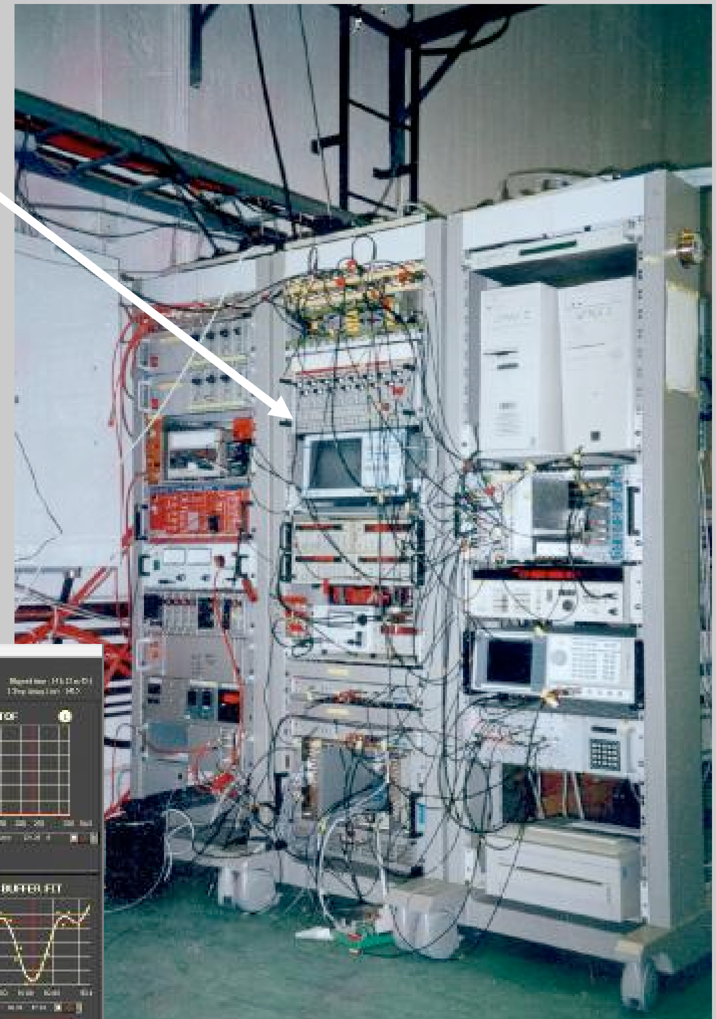
Fällan 10^{-12} mbar

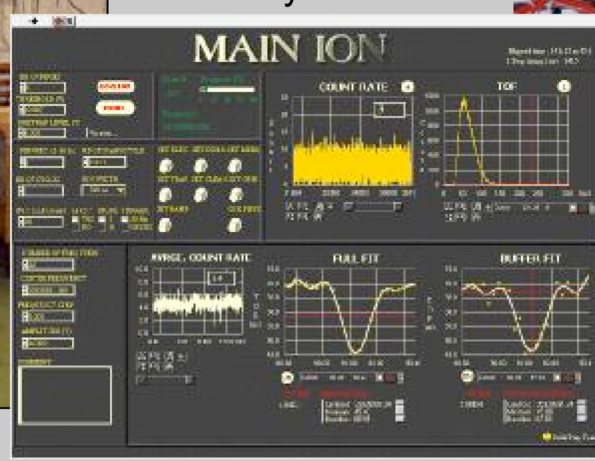
Elektronik-rack

Supraledande magnet inuti en tank med flytande helium

Jontransportsystem vid ultrahögt vakuum $(10^{-9}\,mbar = 10^{-7}\,Pa)$

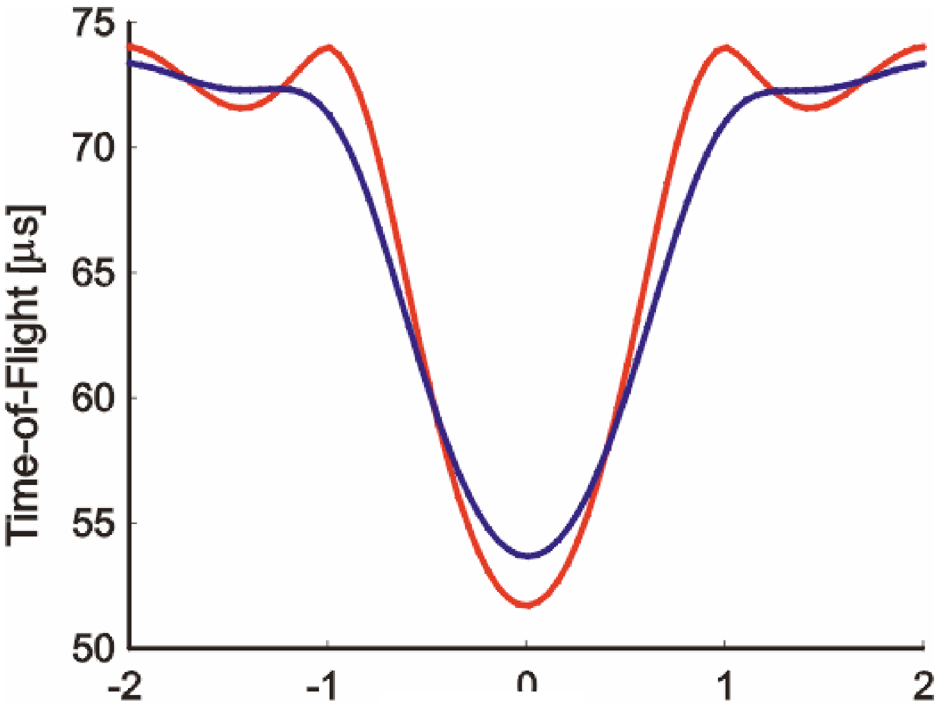
Datoriserat kontrollsystem



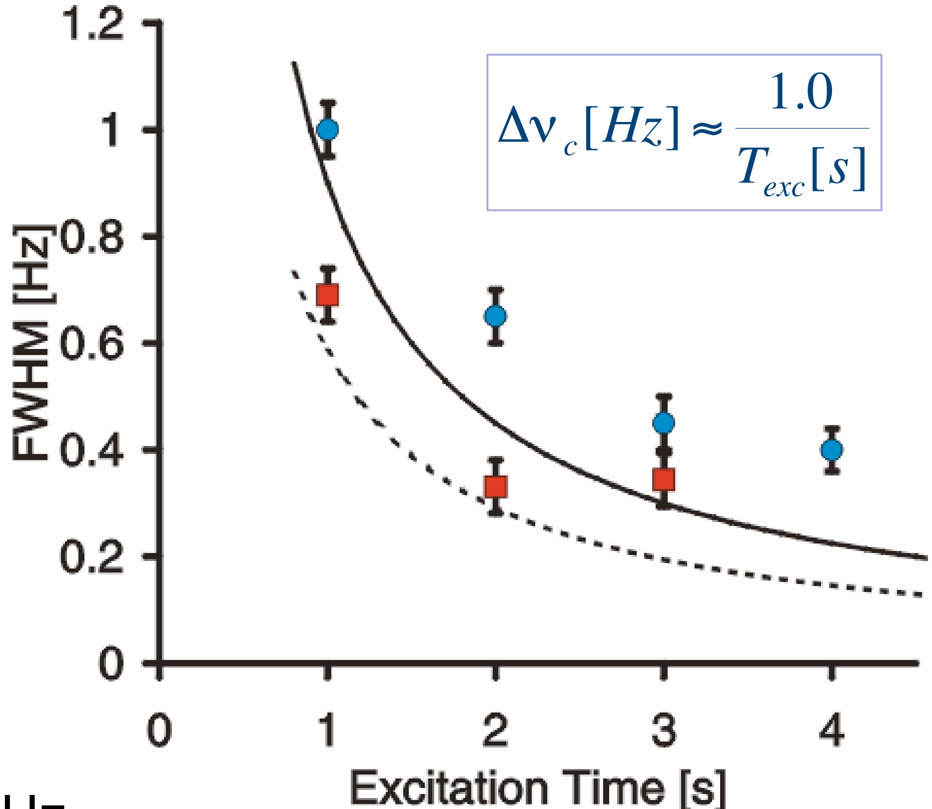


Fourier limit?

resonanskurva



$\nu - \nu_c$ [Hz] $\nu_c \approx 3.5 10^7$ Hz



$$\Delta\nu_c[Hz] \approx \frac{1.0}{T_{exc}[s]}$$

⇒ uplösningen är 0,4 Hz/ 3.5 10^7 Hz ≈ 10^{-8}

precisionen vi kan få med SMILETRAP: $$\frac{m}{\Delta m} = \frac{\nu_c}{\Delta \nu_c} = 10^{10}$$

Tänkvärt: Att mäta med en noggrannhet på **10^{-9}** motsvarar tex:
Att räkna befolkningen i Kina på en person när.
Att mäta avståndet mellan Stockholm och Kiruna på 1 mm när.

Systematical uncertainties

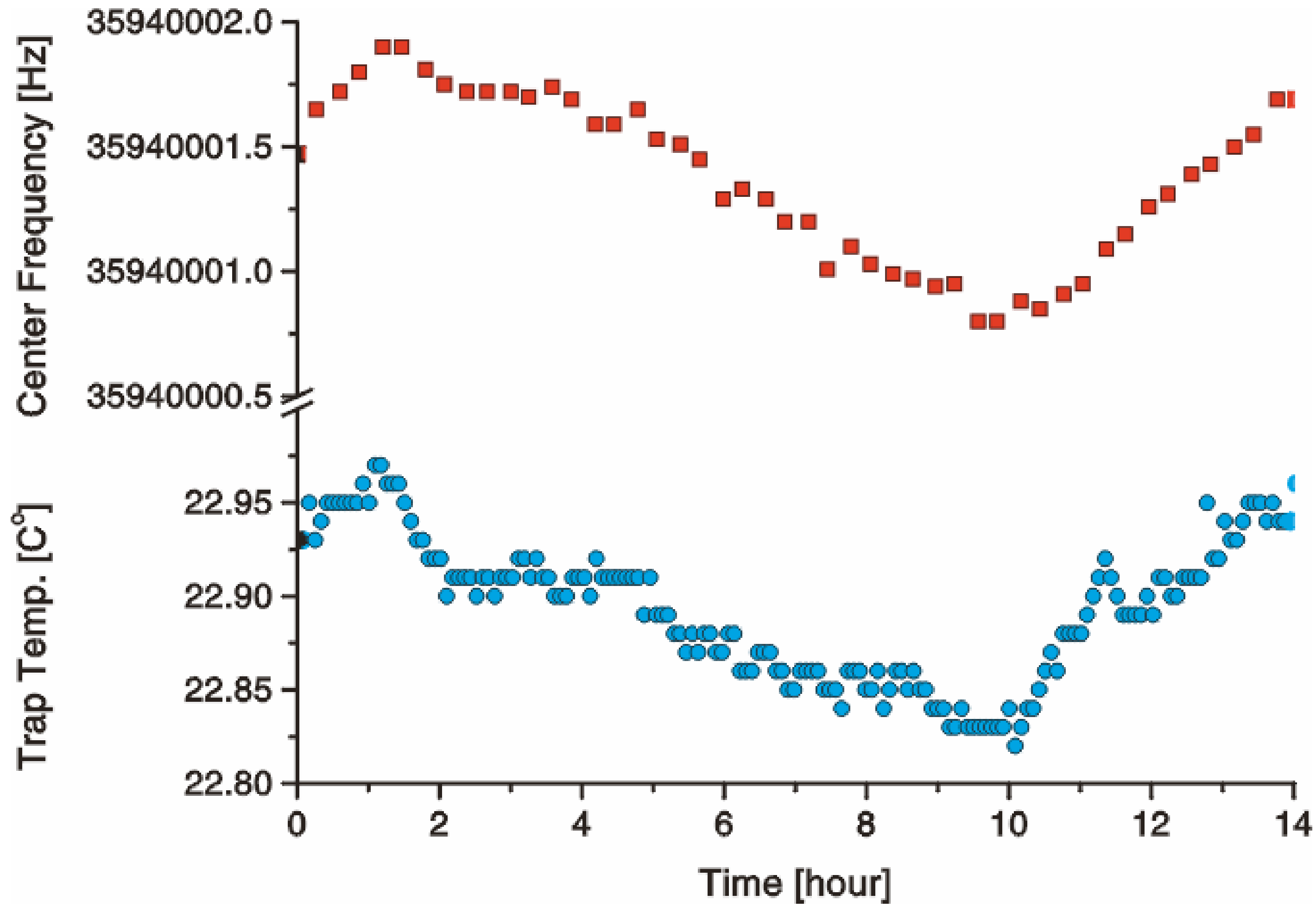
- Uncertainty of the reference mass

  $^{12}C^{q+}$ or H_2^+ as mass references! 0 or 0.14 ppb

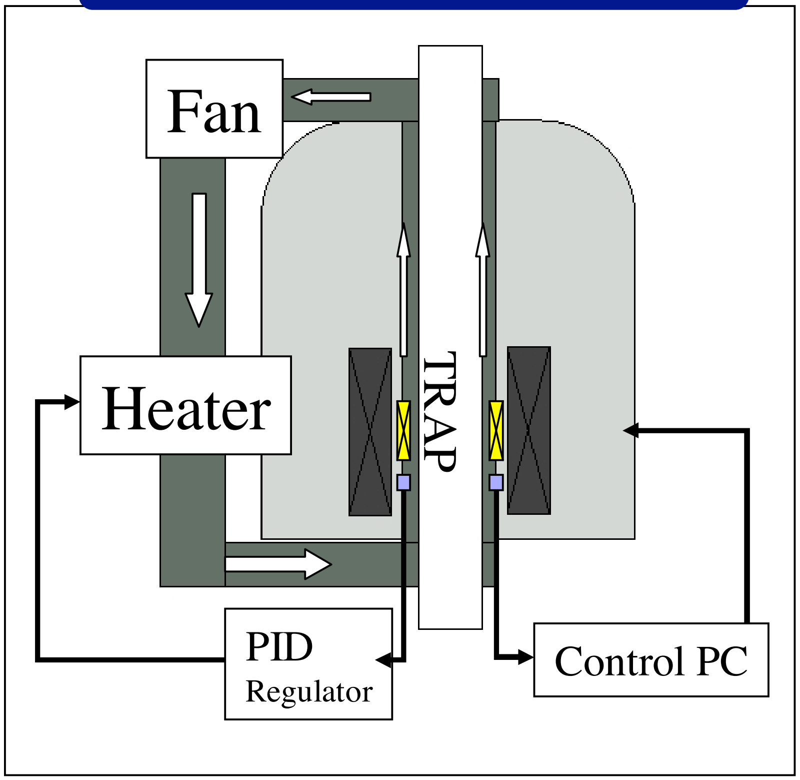
- Field errors, geometrical imperfections and magnetic field fluctuations

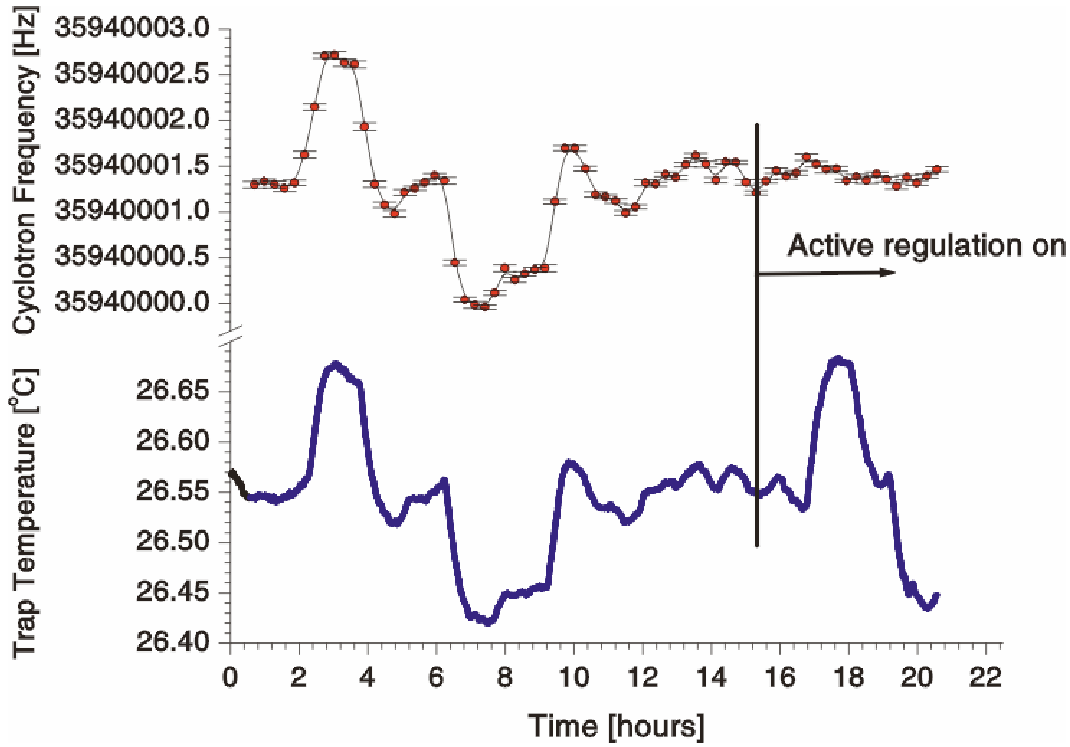
  large trap dimensions, strong magnetic field, small motion amplitudes,a large number of ions, *T* and *p* stabilization. 0.1~ppb

Cyclotron Frequency Fluctuations



T stabilization system





drift **ΔT « 0.01 °C / 24 h**

add. B-field correction

Systematical uncertainties

•Uncertainty of the reference mass

$^{12}C^{q+}$ or H_2^+ as mass references! 0 or 0.14 ppb

•Field errors, geometrical imperfections and magnetic field fluctuations

large trap dimensions, strong magnetic field, small motion amplitudes,a large number of ions, *T* and *p* stabilization. 0.1~ppb

•*q*/*A* dependence and number dependence

take mass doublets and only one ion in the trap

0 to 1 ppb, <1 ppb shift which results in an uncertainty <0.5 ppb

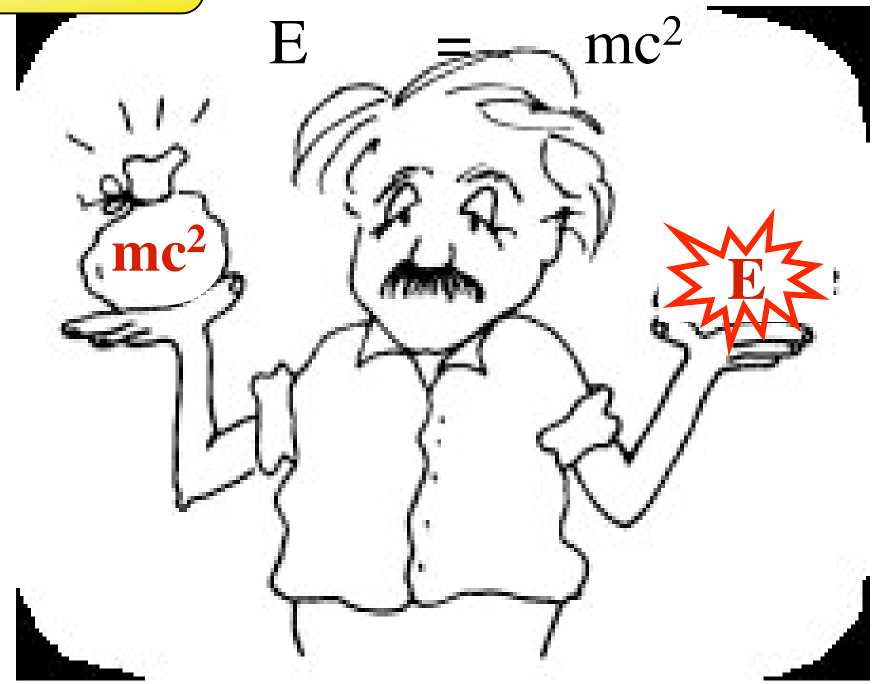
•Measured statistical uncertainty

limits the knowledge of systematic effects

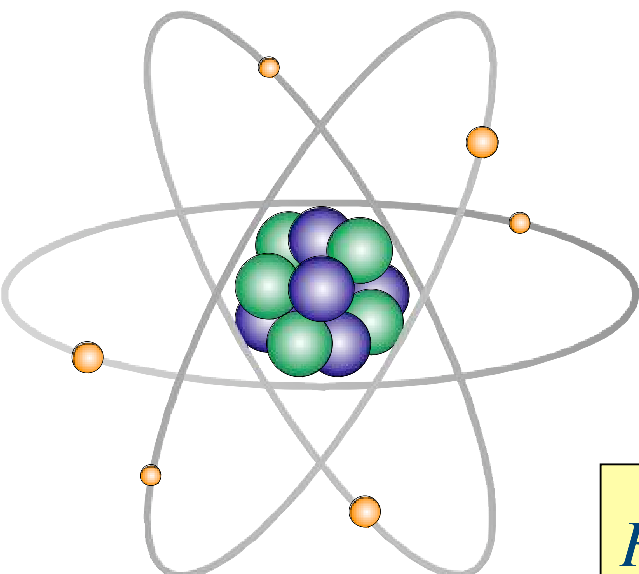
limited by time, number of ions and by field oscillations

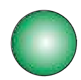
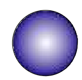
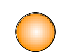
Från jonmassa till atommassa:

Einstein 1905:



Noggranna massbestämningar av atomer ger information om bland annat atomära och nukleära bindningsenergier.



= $N \cdot$  $+ Z \cdot$  $+ Z \cdot$ 

– binding energy

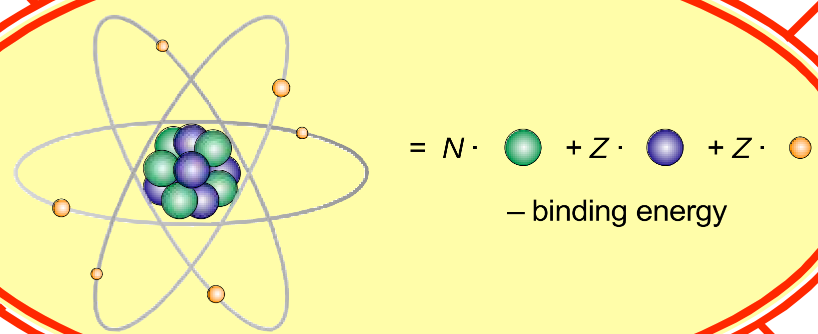
$$R = \frac{\nu_c}{\nu_{cREF}} = \frac{q m_{REF}}{q_{REF} m} \Rightarrow M_A = \frac{1}{R}\frac{q}{q_{REF}} m_{REF} + q m_{e^-} - E_B(A^{q+})$$

The Importance of Atomic Masses

Physics & Chemistry
basic information required
$\delta m/m \approx 1\cdot10^{-5}$

General Physics
fundam. constants
test of CPT
$\delta m/m \leq 1\cdot10^{-10}$

Atomic Physics
binding energy,
QED in HCI
$\delta m/m \leq 1\cdot10^{-9}$



Nuclear Physics
mass formula,
models, halo
$\delta m/m \approx 1\cdot10^{-7}$

Astro-physics
nuclear synthesis,
r-, rp-process
$\delta m/m < 1\cdot10^{-7}$

Weak Interactions
symmetry tests,
CVC hypothsis
$\delta m/m < 3\cdot10^{-8}$

Krav på Noggranhet

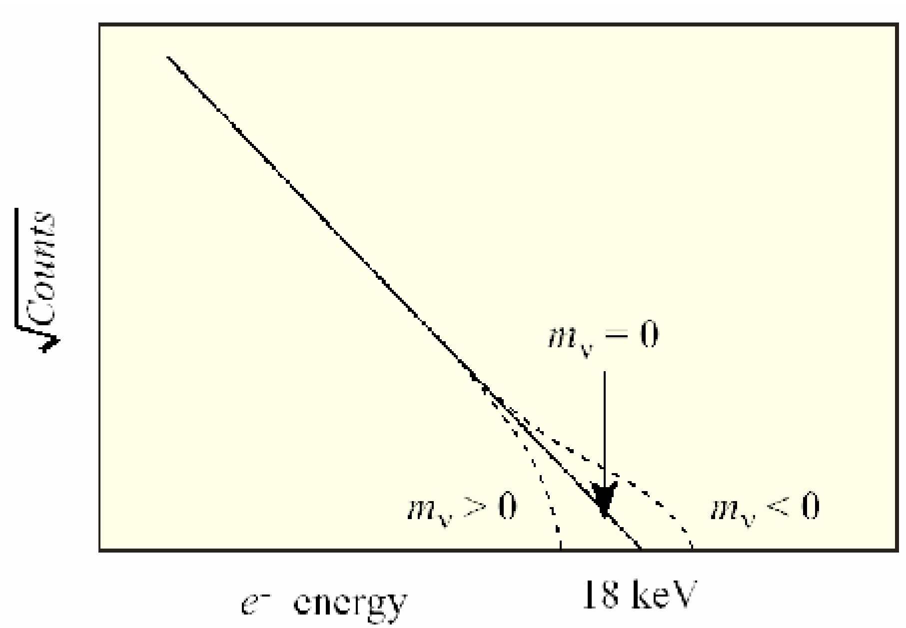
Field	Mass uncert. $\delta m/m$
General physics and chemistry	10^{-6} (ppm)
Nuclear physics • Decay energies • Binding energies	10^{-7}
Nuclear structure, models and formulas • Shell closure, pairing, deformation, halos • Nucleosynthesis, IMME	10^{-7} - 10^{-8}
Fundamental studies with radionuclides • Symmetry tests • Weak interaction studies (CVC hypothesis)	10^{-8} - 10^{-9} (ppb)
Fundamental studies with stable nuclides • Binding energies, QED in HCl • Fundamental constants, test of CPT	10^{-9} - 10^{-11}

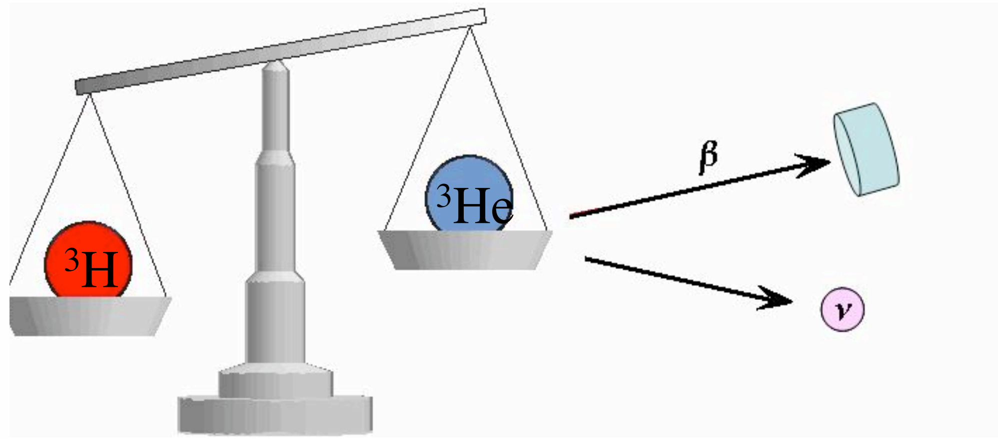
SMILETRAP

SMILETRAP Exempel:

- Si-massa för ny Kilogram definition, ✓
- ^{3}H - ^{3}He massa för neutrinomassa från β sönderfall,
- Cs massa för ny finstruktur-konstant,
- Ca massa för elektronens g-faktor

Hur kan neutrino massan bestämmas





Tritiums beta sönderfall *Q*-värde

$$^3_1H \rightarrow ^3_2He + e^- + \bar{\nu}$$

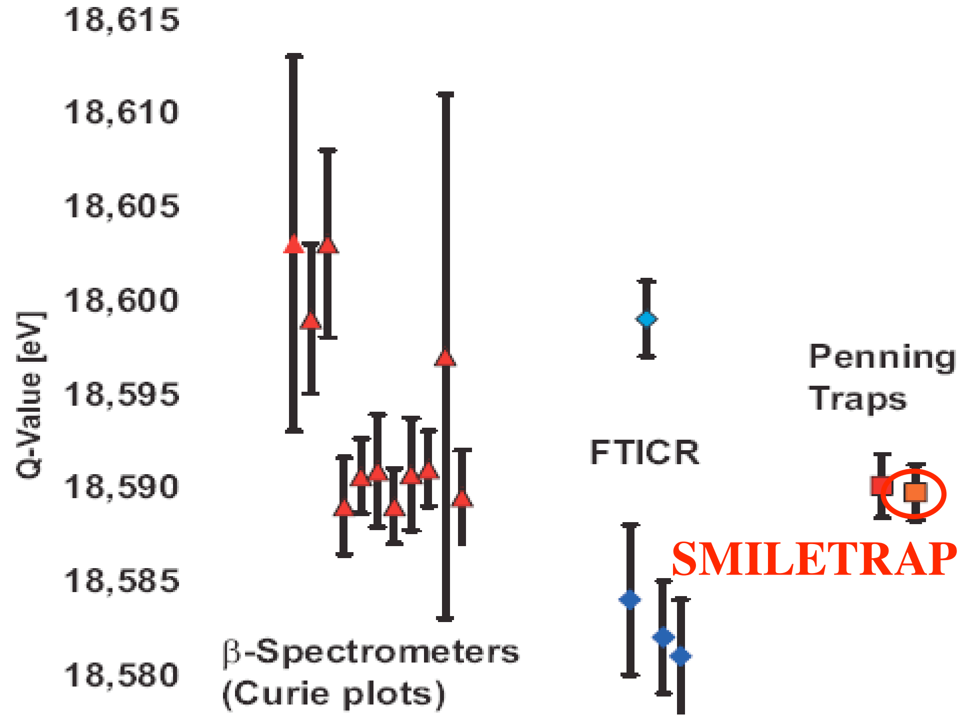
Q=18,589 8(12) keV

Skillnaden i massa mellan

$^3H^{1+}$ - $^3He^{1+}$

Bestämd <1eV ($<3\cdot10^{-10}$)

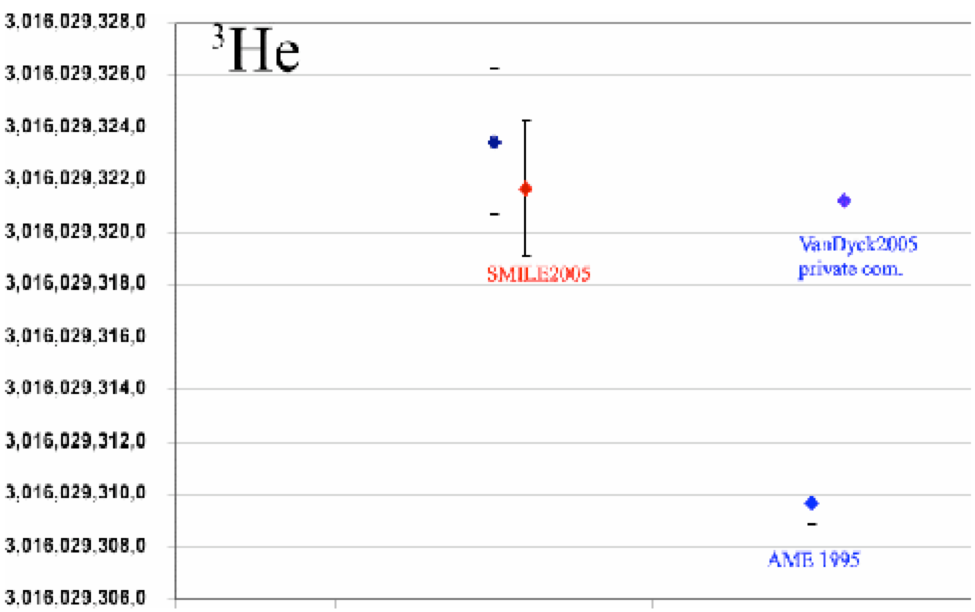
KATRIN ~30 meV ($\Delta m/m$ 10^{-11})

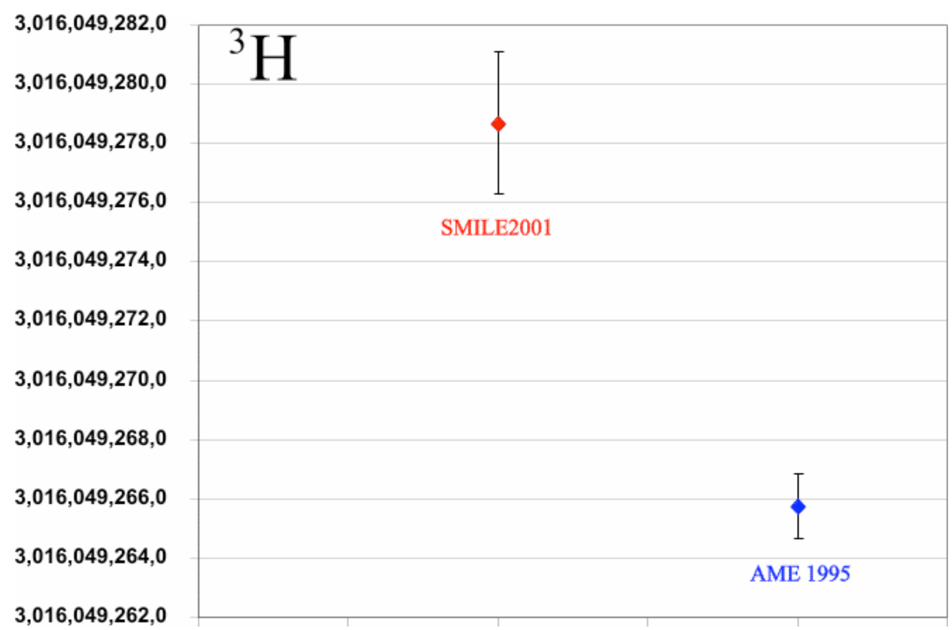


^{3}He mass result-2005

^{3}He	3,016,029,321 (26) u	0.8 ppb total

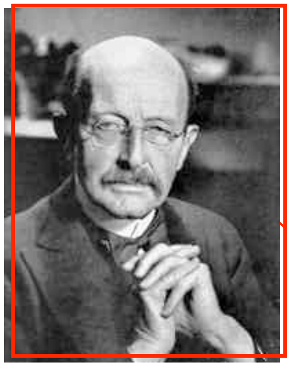
^{3}H	3,016,049,278 (24) u	0.8 ppb total





In classical electrodynamics interaction between point charges $Q_1 \xrightarrow{\mathbf{r}} Q_2$ described by $\mathbf{F}(r) = -\,\mathrm{grad}V(r)$, $V(r) = Q_1 Q_2 / 4\pi\epsilon r$ (analog for charge moving in magnetic field)

Maxwell



Planck

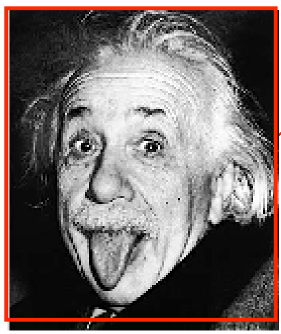
quantization of fields →

Charged particles → field quanta are photons

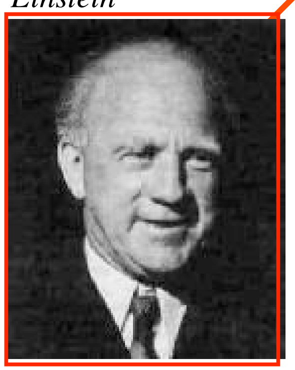
electromagnetic interaction = exchange of virtual photons $h\nu$

$Q_1 \xrightarrow{h\nu} Q_2$

$h\nu$ has energy and momentum within conditions given by uncertainty relation $\Delta E\, \Delta t = h$, $\Delta p\, \Delta x = h$ (virtual photons)



Einstein



Heisenberg

One photon exchange dominant in Coulomb force ⇒

(for $m\gamma = 0$) $\Delta p = E/c = h/c\, \Delta t$; $\Delta t = r/c$ ⇒

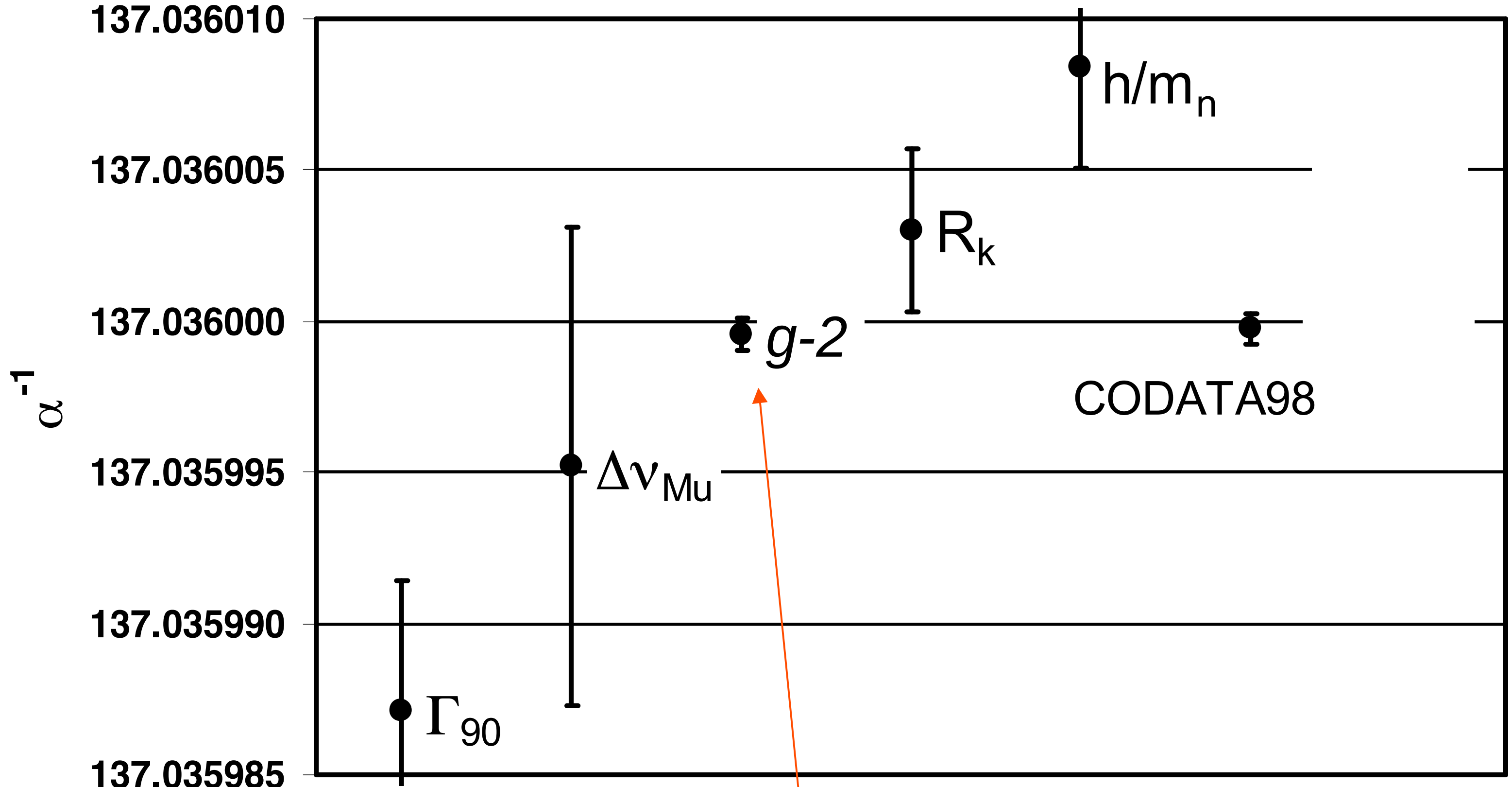
$F(r) = \Delta p / \Delta t \sim h/c\, \Delta t^2 = hc/r^2$

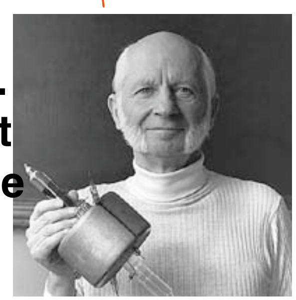
by comparison with $V(r)$ ⇒ proportionality constant

$Q_1 Q_2 / 4\pi\epsilon hc = n_1 n_2 e2/4\pi\epsilon hc =$ **α fine structure constant ≈ 1/137** (constant of el.magn. interaction, time dependent?)

consequent ⇒ Quantum Electro-Dynamics (QED).

Different determinations of the Fine Structure Constant α





Hans G. Dehmelt Nobelprize 1989

^{133}Cs for Accurate Determinations of the **Fine Structure Constant** α

$$\alpha^2 = \left(\frac{2R_\infty}{c}\right)\left(\frac{h}{m_e}\right) = \left(\frac{2R_\infty}{c}\right)\left(\frac{h}{m_{Cs}}\right)\left(\frac{m_{Cs}}{m_p}\right)\left(\frac{m_p}{m_e}\right)$$

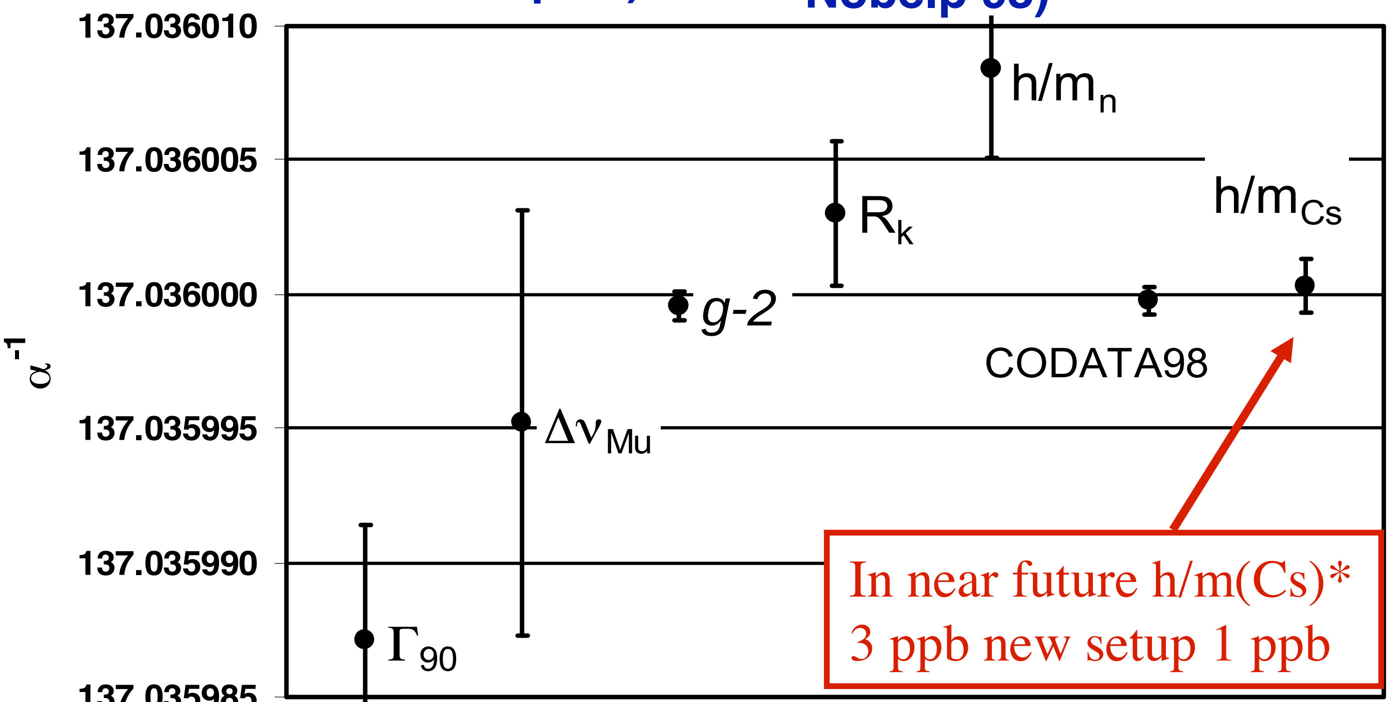
2.2 ppb (vanDyck)

< 2·10^{-11} (Hänsch Nobelp 05)

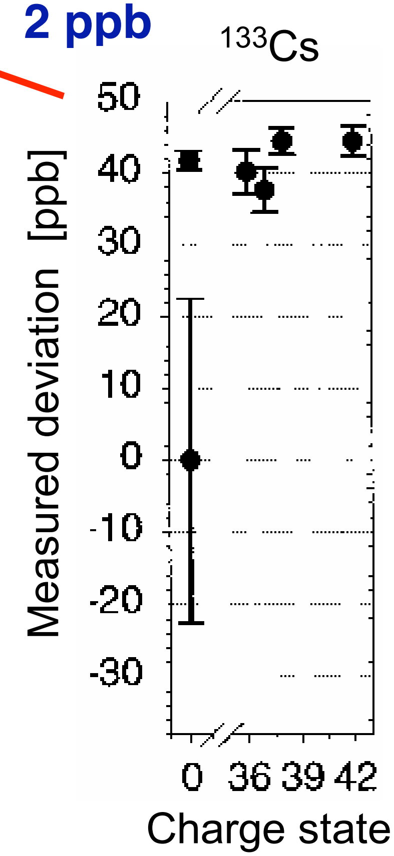
7.3 ppb (S.Chu Nobelp 98)

SMILETRAP

2 ppb



*S. Chu, private communication (2003)



PRL 83, 1999

QED (QuantumElectroDynamics)

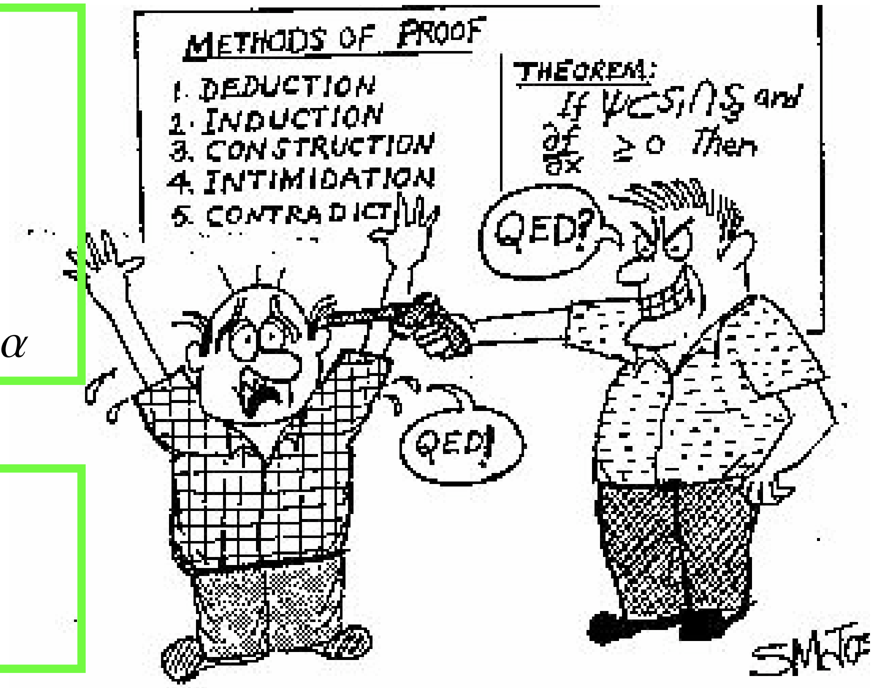
$$\vec{\mu} = -g \frac{e}{2m_e} \vec{J}$$

μ= magnetiskt moment
e, m_e elektronens laddning och massa
J =rotationsmoment
g =g-faktorn

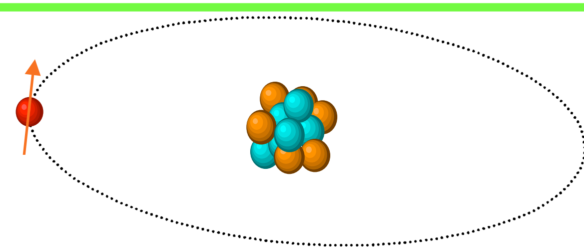
Enligt Diracs relativistiska kvantmekanik är g = 2 för en **fri elektron**.

QED-beräkningar ger i stället g = 2.0023...

Används för definition av Finstrukturkonstanten α



Tittar på **bunden electron**:
Genom att experimentellt bestämma g med stor noggrannhet kan QED testas.

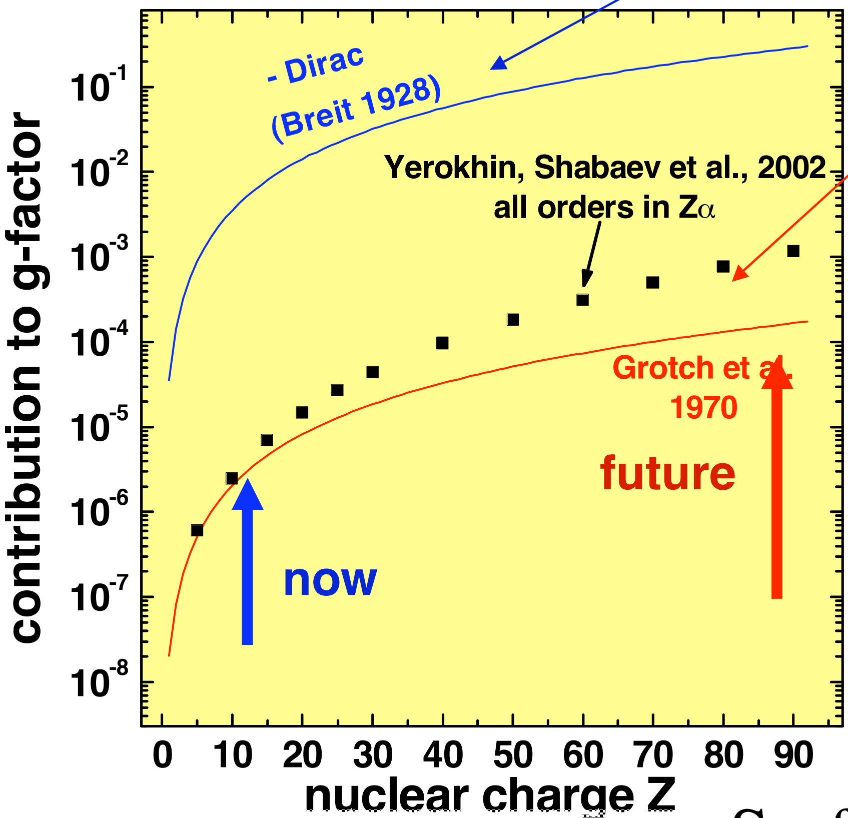


Test av QED: Bundna elektronens g-faktor

$$g_{bound}/g_{free} \approx 1 - 1/3(Z\alpha)^2 + \alpha/4\pi(Z\alpha)^2$$

relativistic effect (Dirac)

bound-state QED



Precise g-Factor measurements @ HITRAP(GSI) & SMILETRAP

Larmor precesion

$$h\nu_L = g_J \mu_B B$$

Cyclotron motion

$$\nu_c = \frac{q}{2\pi M_{ion}} B$$

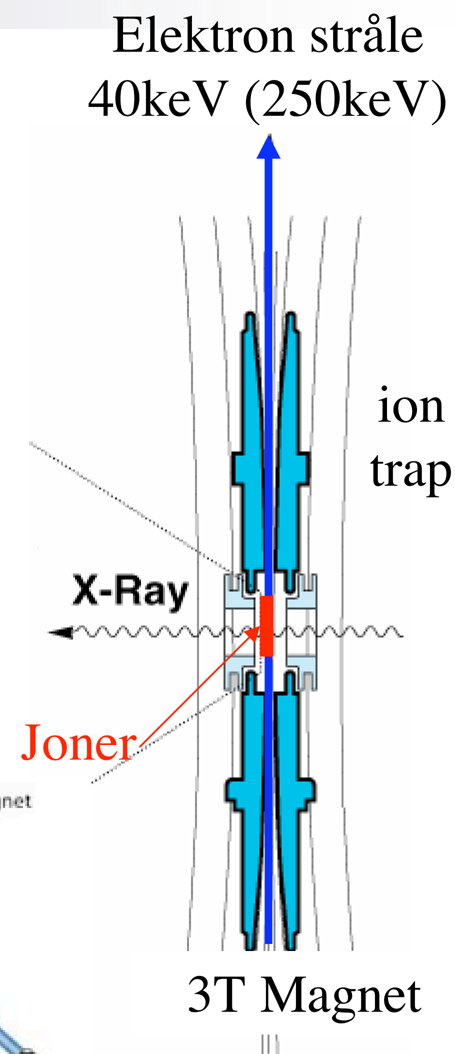
$$\Rightarrow \quad g_J = \frac{2\omega_L}{\omega_c^{ion}} \frac{q m_e}{M^{ion}}$$

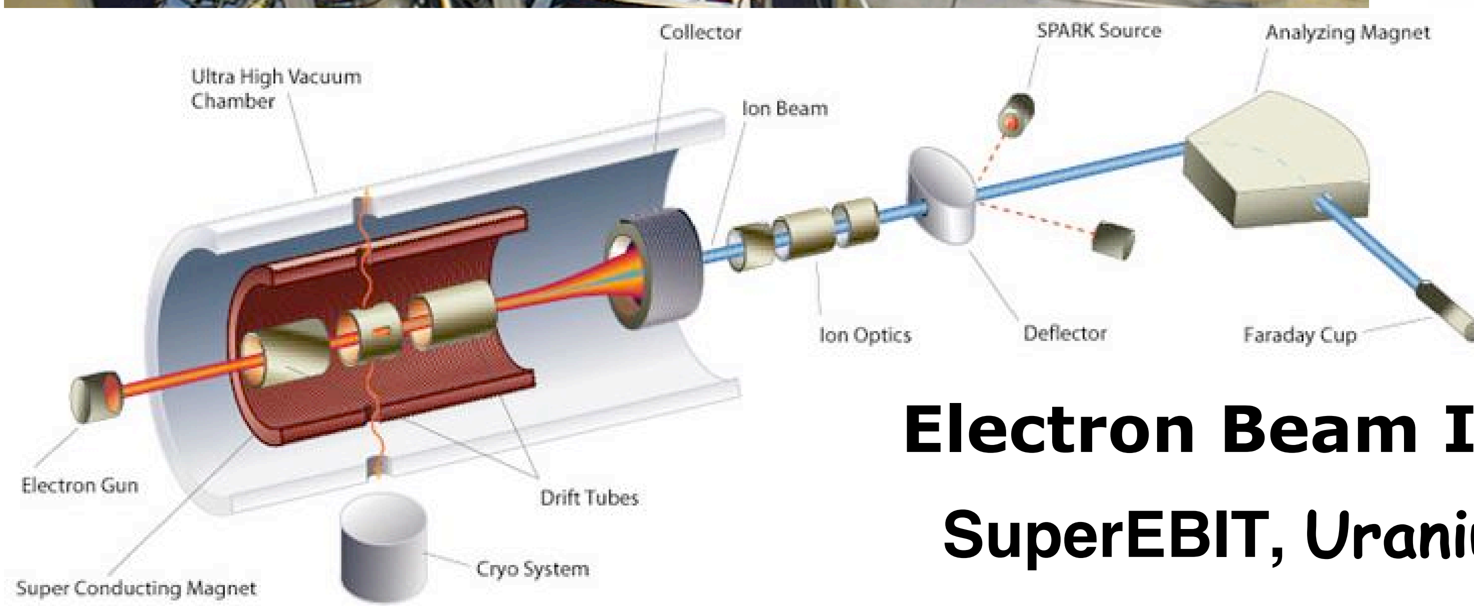
So far results for C^{5+}, O^{7+}, next $^{40}Ca^{19+}$

Framtiden: need mass of $Ca^{19+} \rightarrow U^{91+}$ with <0.5 ppb from SMILETRAP

Nytt lab på AlbaNova

Extremt högt laddade joner





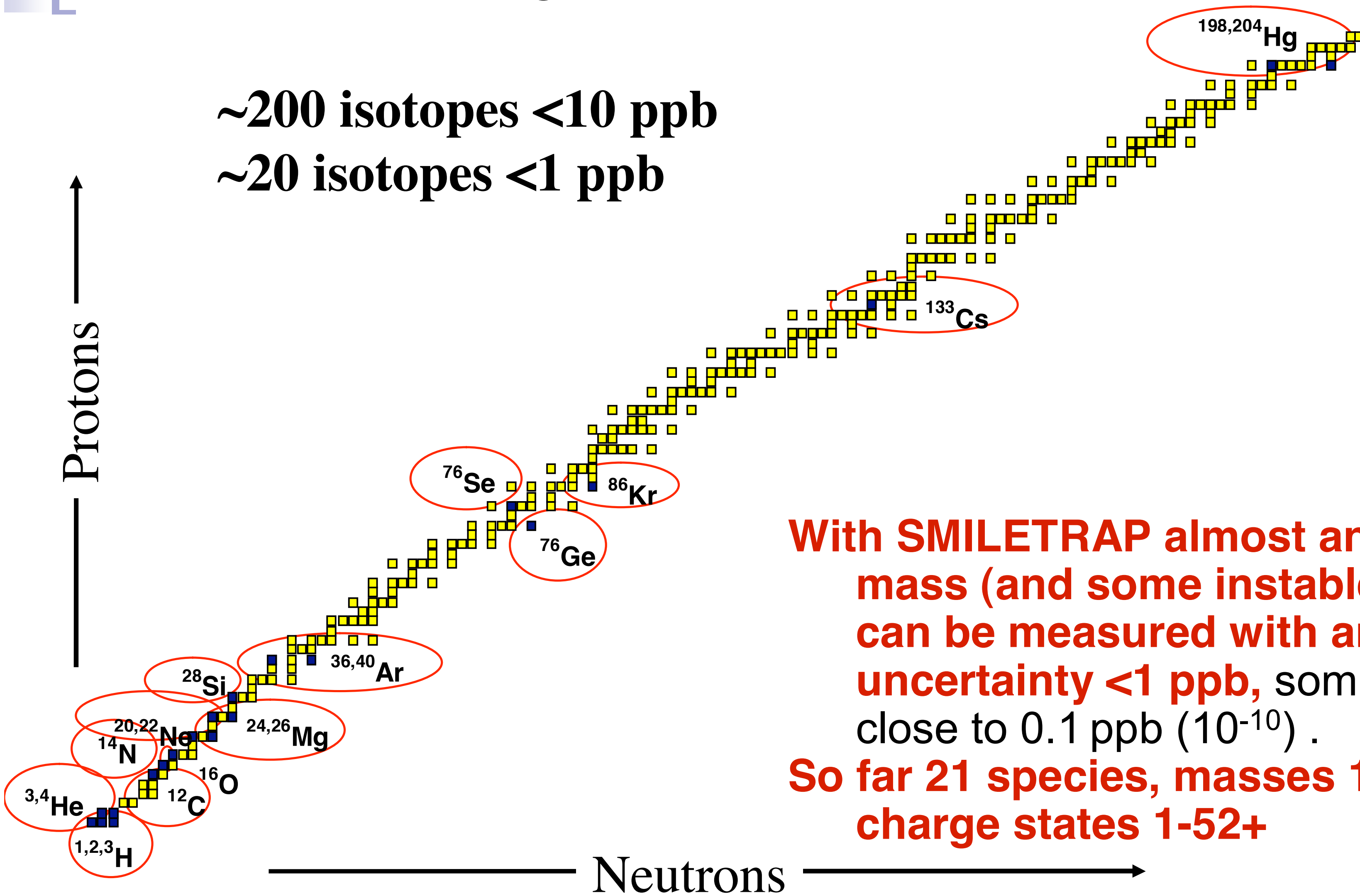
Electron Beam Ion Trap

SuperEBIT, *Uranium 92+*

Current knowledge of atomic masses

~200 isotopes <10 ppb

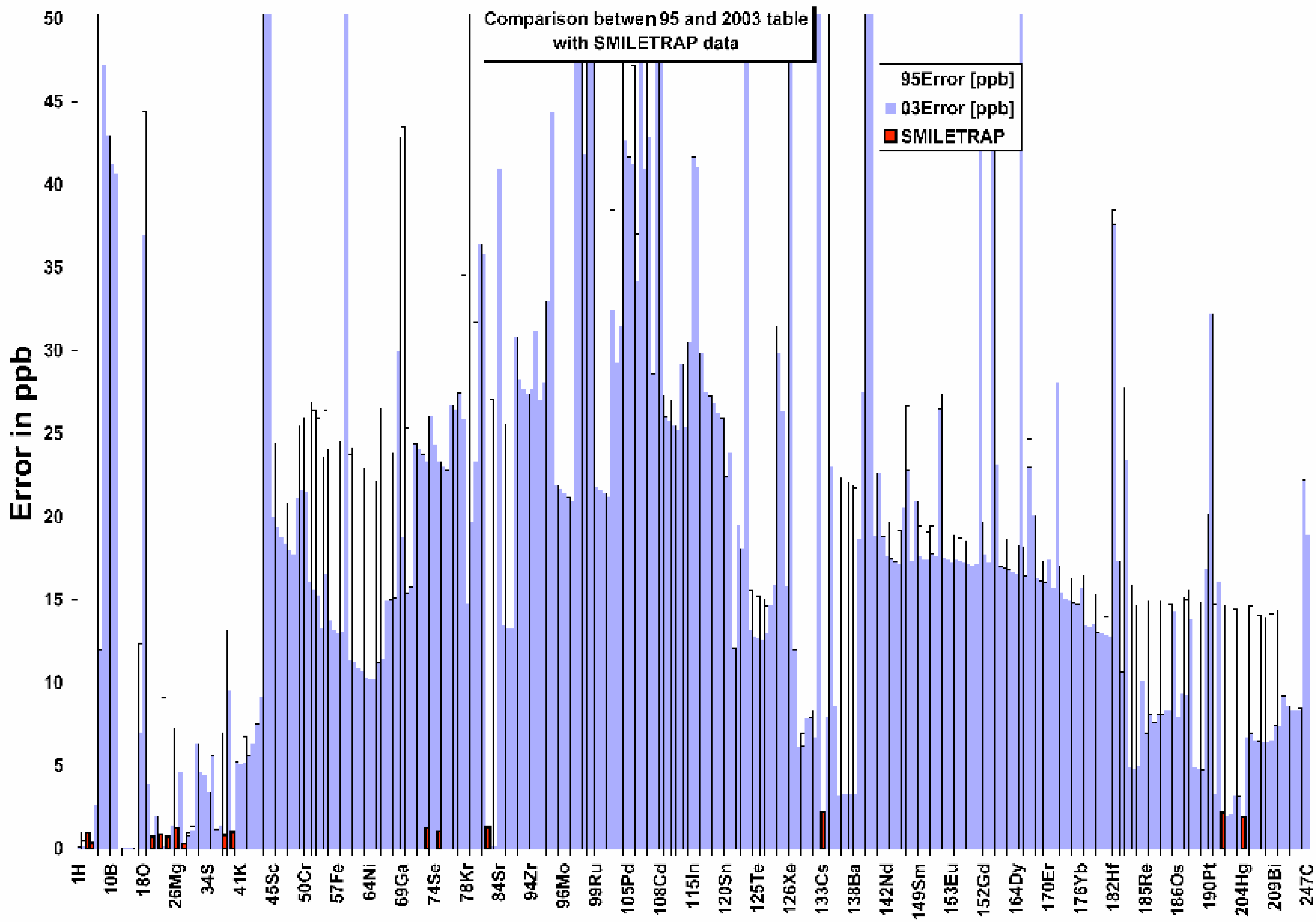
~20 isotopes <1 ppb



With SMILETRAP almost any stable mass (and some instable masses) can be measured with an uncertainty <1 ppb, sometimes close to 0.1 ppb (10^{-10}) .

So far 21 species, masses 1-204u, charge states 1-52+

Previous SMILETRAP highlights



New masses of atoms and where it matters...

- ^{76}Ge-^{76}Se for constraints on neutrino-less double beta decay

- Heavy M_N to solve the ”$^{198-204}$Hg problem in Audi & Wapstras mass table
- Weighing atomic binding energies ⇒ Ionization potentials of ions
- Mass to energy conversion ⇒ Relativity Test $E=mc^2$

……

… a relative mass accuracy of $\delta m/m = 10^{-11}$ is the goal!